Oscillations

Oscillations

- pendulum
- vibration of a guitar string
- vibration of conduction

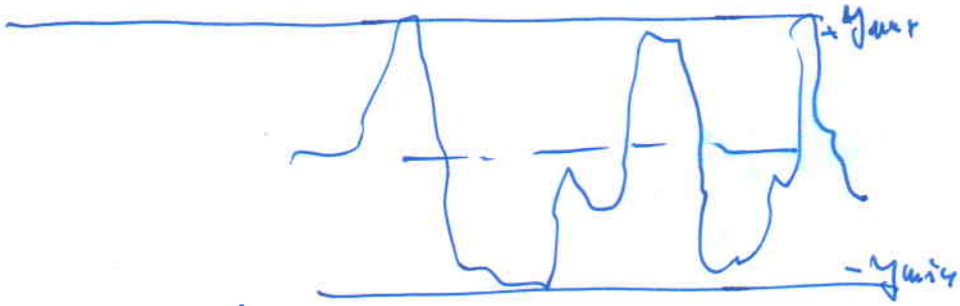
mechanics oscillations

- electromagnetic oscillations

- Mathematic description is the same in all cases.

Oscillations

$$y(t) \le |y_{max}(t)| \quad \text{band}$$



Oscillations are periodic if

$$y(t) = y(t+T) = y(t+kT)$$

T .. [s] --- period

$f = \frac{1}{T}\ [s^{-1} = Hz]$ hertz frequency

Oscillations
- Undamped = free = idealization
  netlumené
- Damped = Real (with friction)
  Tlumené

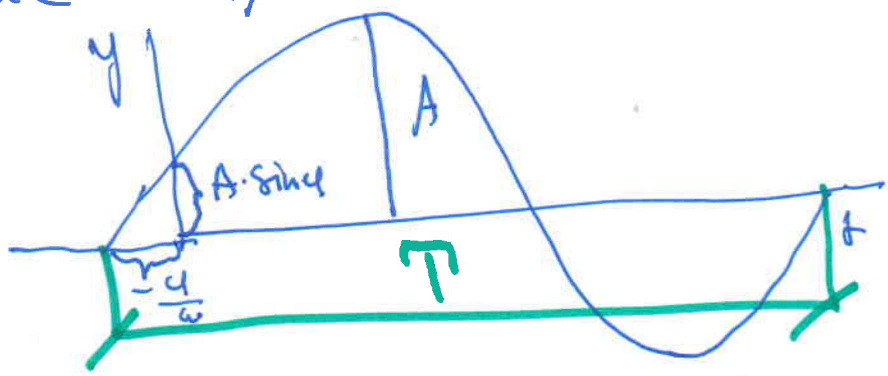
40. Free harmonic oscillations

Free = undamped

harmonic = sin / cos

oscillations – $y(t) \leq |y_{max}(t)|$

$$y = A \cdot \sin(\omega t + \varphi)$$



y - position

A - amplitude

$(\omega t + \varphi)$ - phase of the oscillations
[feiz]

φ initial phase

ω ... angular frequency $[s^{-1}]$

$$\omega t + \varphi + 2\pi = \omega(t + T) + \varphi$$

$$2\pi = \omega T$$

$$\boxed{\omega = \frac{2\pi}{T} = 2\pi f}$$

Position

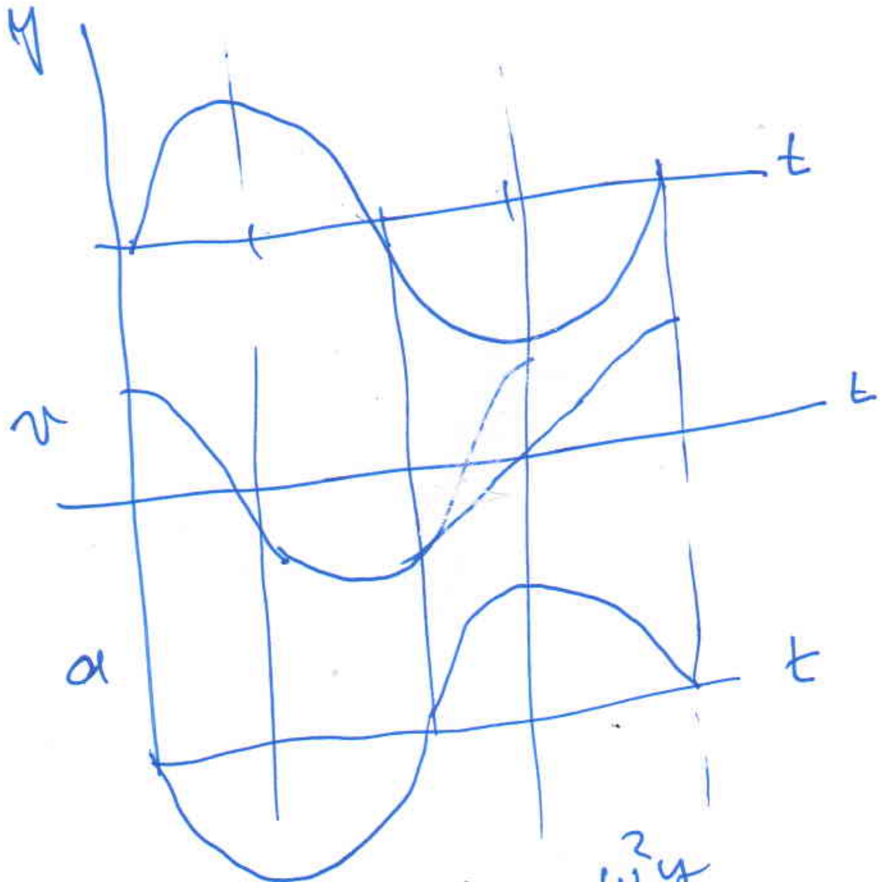
$y = \underbrace{A \cdot \sin(\omega t + \varphi)}$

Velocity $- v = \frac{dy}{dt} = \omega A \cdot \cos(\omega t + e)$

acceleration $a = \frac{dv}{dt} = -\omega^2 \underbrace{A \cdot \sin(\omega t + \varphi)}$

$\boxed{a = -\omega^2 y}$

The acceleration is always proportional to the displacement, but oppositely directed.



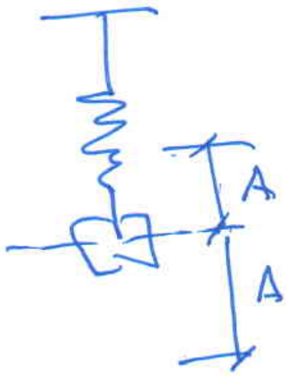
$a = -\omega^2 y$

$y'' = -\omega^2 y$

$\boxed{y'' + \omega^2 y = 0}$

Differencial eq. for free harmonic oscillations

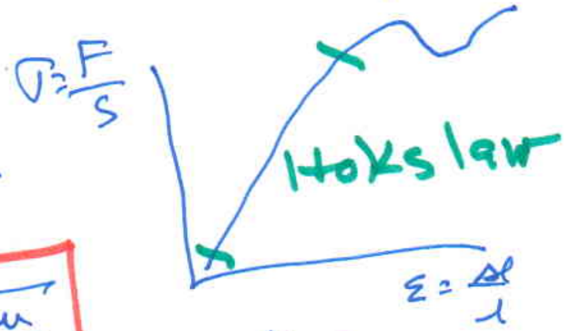
Dynamic view



$$\boxed{F = -ky}$$ Force eq.

$$ma = -ky \quad \Big|\ \cdot\frac{1}{m}$$

$$y'' + \underbrace{\frac{k}{m}}_{\omega^2} y = 0$$



$$\omega^2 = \frac{k}{m} = \left(\frac{2\pi}{T}\right)^2$$

$$\boxed{T = 2\pi\cdot\sqrt{\frac{m}{k}}}$$ — Doesn't depend on the amplitude A

$$\boxed{y'' + \omega^2 y = 0}$$ Dif. eq. which describe free harmonic osc.

Solution

$$\boxed{y = A\cdot\sin(\omega t + \varphi)}$$

41. Energy of harmonic oscillations

Kinetic energy

$$E_k = \frac{1}{2} m v^2$$

$$E_k = \frac{1}{2} m \omega^2 A^2 \cdot \cos^2(\omega t + \varphi)$$

$$y = A \cdot \sin(\omega t + \varphi)$$

$$v = \frac{dy}{dt} = \omega A \cdot \cos(\omega t + \varphi)$$

$$a = \frac{dv}{dt} = -\omega^2 A \sin(\omega t + \varphi)$$

$$a = -\omega^2 y$$

Potential energy

$$E_p = W = -\int_0^y F dy = \dots = -\int_0^y m a dy =$$

$$E_p = +\int_0^y m \cdot \omega^2 y dy = m \omega^2 \left[\frac{y^2}{2}\right]_0^y =$$

$$E_p = \frac{1}{2} m \omega^2 \cdot A^2 \cdot \sin^2(\omega t + \varphi)$$

$$E_c = E_k + E_p$$

$$E_c = \frac{1}{2} m \omega^2 A^2 \cos^2(\omega t + \varphi) + \frac{1}{2} m \omega^2 A^2 \cdot \sin^2(\omega t + \varphi)$$

$$E_c = \frac{1}{2} m \omega^2 A^2 [\underbrace{\cos^2(\omega t + \varphi) + \sin^2(\omega t + \varphi)}_{1}]$$

$$\boxed{E = \frac{1}{2} m \cdot \omega^2 A^2 = \text{constant}}$$

$\rightarrow$ There is not time.

42. Damped oscillations

Free oscillations = is idealization [aidielaizeišn]

Damped osc. = real ⇒ - friction < in spring / in surroundings of the body

⇒ reducing its amplitude

$$\boxed{F_t = -R_m \cdot v}$$

F_t = damping force

R_m - mechanics resistance

$$\boxed{F = -R_m \cdot v - ky}$$

Force eq.

$$ma + R_m v + ky = 0$$

$$m y'' + R_m \cdot y' + ky = 0 \quad \Big| \cdot \frac{1}{m}$$

$$y'' + \overbrace{\frac{R_m}{m}}^{2\delta} y' + \overbrace{\frac{k}{m}}^{\omega^2} y = 0$$

$$\boxed{y'' + 2\delta y' + \omega^2 y = 0}$$

Dif. eq. which describes damped oscillations

The solution $y = C \cdot e^{\alpha t}$

$$\alpha^2 + 2\delta\alpha + \omega^2 = 0 \quad \text{characteristic eq.}$$

$$ax^2 + bx + c = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\alpha_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega^2}$$

$$\alpha_1 = -\delta + j \cdot \omega_1$$

$$\alpha_2 = -\delta - j\omega_1$$

$$\sqrt{-1} = j$$

$$\delta^2 - \omega^2 = \omega_1^2$$

$$y = C_1 \cdot e^{\alpha_1 t} + C_2 e^{\alpha_2 t}$$

$$y = c_1 \cdot e^{\alpha_1 t} + c_2 e^{\alpha_2 t}$$

$$e^{\pm j\omega_1 t} = \cos\omega_1 t \pm j\sin\omega_1 t$$

1, $\delta^2 - \omega^2 < 0 \Rightarrow \delta < \omega$

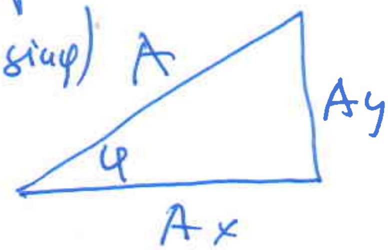
$$y = e^{-\delta t}\left[c_1\cos\omega_1 t + j c_1 \sin\omega_1 t + c_2\cos\omega_1 t - j c_2 \sin\omega_1 t\right] =$$

$$= e^{-\delta t}\left[\underbrace{j(c_1 - c_2)}_{A_x}\sin\omega_1 t + \underbrace{(c_1 + c_2)}_{A_y}\cos\omega_1 t\right] =$$

$c_1, c_2 \ldots$ integrations constants

$$y = e^{-\delta t}\left(A_x \cdot \sin\omega_1 t + A_y \cdot \cos\omega_1 t\right)$$

$$y = e^{-\delta t}\left(A \cdot \cos\varphi \cdot \sin\omega_1 t + A \cdot \cos\omega_1 t \cdot \sin\varphi\right)$$



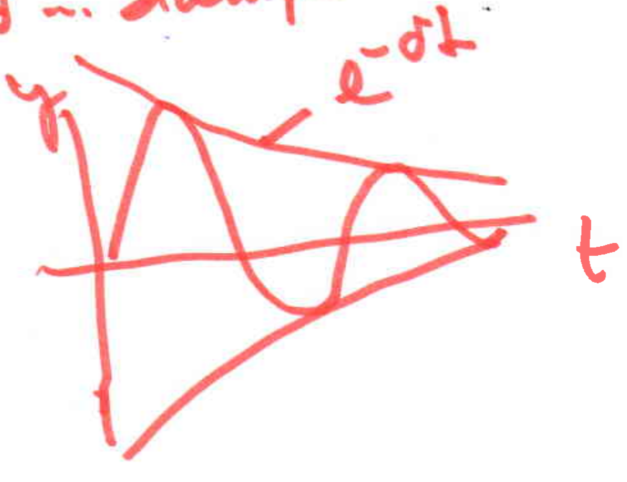
$$\boxed{y = A \cdot e^{-\delta t} \sin(\omega_1 t + \varphi)}$$

$\cos\varphi = \frac{A_x}{A}$

$\sin\varphi = \frac{A_y}{A}$

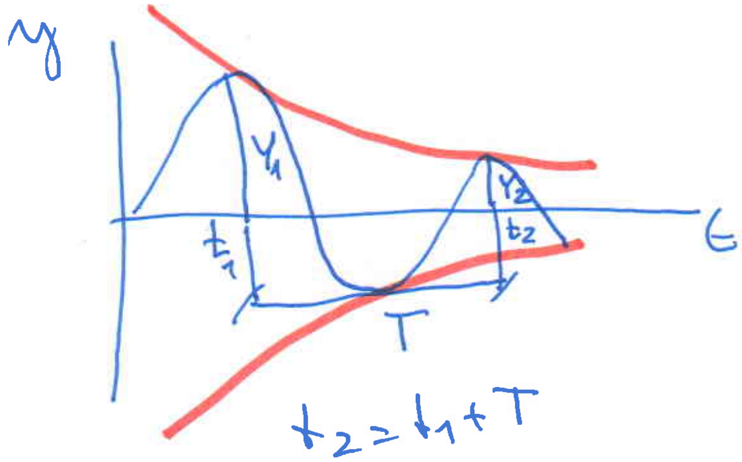
for $\delta < \omega$

δ ... damped constant



$$\boxed{b = \frac{y_1}{y_2}}$$

b … decrement útlmu



$$t_2 = t_1 + T$$

$$b = \frac{y_1}{y_2} = \frac{A \cdot e^{-\delta t_1} \cdot \sin(\omega_1 t_1)}{A \cdot e^{-\delta t_2} \cdot \sin(\omega_1 t_2)} =$$

$$b = \frac{\cancel{A} \cdot \cancel{e^{-\delta t_1}} \cdot \cancel{\sin}(\omega_1 t_1)}{\cancel{A} \cdot e^{-\delta(\cancel{t_1}+T)} \cdot \cancel{\sin}(\omega_1(t+T))} = e^{\delta T}$$

$$\boxed{b = e^{\delta T}}$$ decrement

ϑ … logarithmic decrement

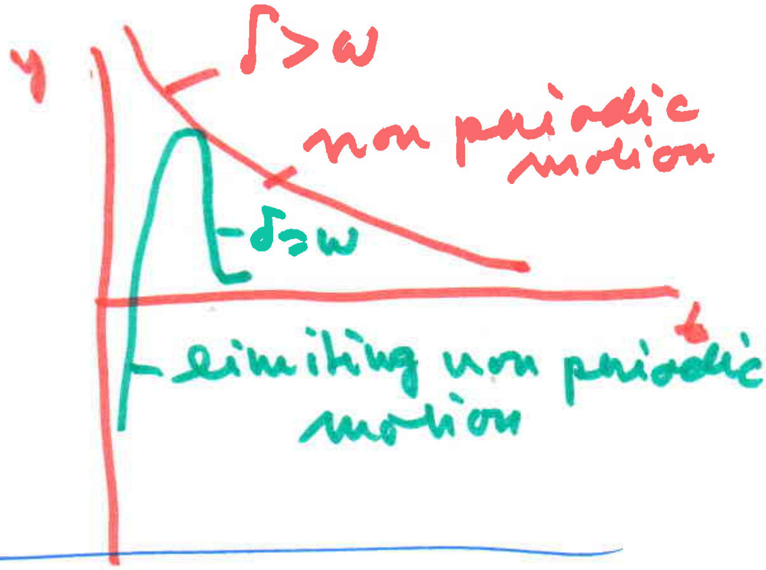
$$\boxed{\vartheta = \ln b = \delta T}$$

2) $\delta^2 - \omega^2 > 0 \Rightarrow \delta > \omega$

$\Rightarrow$ two differents roots

$$y = C_1 e^{\beta_1 t} + C_2 e^{\beta_2 t}$$

cannot be converted to < sin or cos



3) $\delta^2 - \omega^2 = 0$

$\alpha_{1,2} = -\delta$ — Two identical roots

$$y = C_1 e^{-\delta t} + C_2 t e^{-\delta t} = (C_1 + C_2 t) e^{-\delta t}$$

cannot be convert to < sin cos

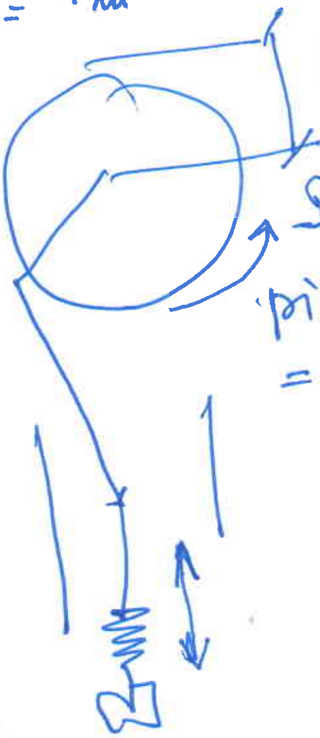
in $t = T$ ... $y = \frac{y}{100}$ 1%

Use < weight maschine, electric equipments

43. Forced oscillations

Excitation force

$$F_b = F_m \cdot \sin(\Omega t)$$



$$\boxed{F = - R_m y' - ky + F_m \cdot \sin(\Omega t)}$$

Force equations

$$m y'' + R_m y' + ky = F_m \cdot \sin(\Omega t) \;\Big|\; \frac{1}{m}$$

$$y'' + \overbrace{\frac{R_m}{m}}^{2\delta} y' + \overbrace{\frac{k}{m}}^{\omega^2} y = \overbrace{\frac{F_m}{m}}^{A_V} \sin(\Omega t)$$

$$\boxed{y'' + 2\delta y' + \omega^2 y = A_V \cdot \sin(\Omega t)}$$

Dif. eq. for forced oscillations

Solution $$\boxed{y = \cancel{A \cdot e^{-\delta t} \cdot \sin(\omega t + \varphi)} + A_V \cdot \sin(\Omega t)}$$

after a short time

$$\boxed{y = A_V \cdot \sin(\Omega t)}$$

A_V - amplitude of forced oscillations

From dif. eq.

$$A_v = \frac{\frac{F_m}{m}}{\sqrt{(\omega^2 - \Omega^2)^2 + 4\delta^2\Omega^2}}$$

A_v will be maximal if denominator will be the least.

$$\frac{\partial(\,)}{\partial \Omega} = 0$$

$\Rightarrow$ $\boxed{\text{Resonance}}$ – between own oscillation and periodic force (Ω).

$\Omega = l = c \cdot T = \frac{c}{f}$

Be careful!!

Bridge

1, Volné harmonické kmity
Free harmonic oscil.

$$\boxed{F = -ky}$$ Silová rovnice

$$\boxed{y'' + \omega^2 y = 0}$$ Diferenciální rov.

$$\boxed{y = A \cdot \sin(\omega t + \varphi)}$$ Řešení - výchylka

2, Tlumení kmity Damped oscillations

$$\boxed{F = -R_m \cdot y' - ky}$$ Silová rov.

$$\boxed{y'' + 2\delta y' + \omega^2 y = 0}$$ Dif. rov.

$$\boxed{y = A \cdot e^{-\delta t} \sin(\omega t + \varphi)}$$ $\delta < \omega$ Výchylka

3, Vynucení kmity Forced oscillations

$$\boxed{F = -R_m y' - ky + F_m \cdot \sin(\Omega t)}$$ Silová rov.

$$\boxed{y'' + 2\delta y' + \omega^2 y = A_v \cdot \sin(\Omega t)}$$ Dif. rov.

$$\boxed{y = \cancel{A \cdot e^{-\delta t} \sin(\omega t + \varphi)} + A_v \cdot \sin(\Omega t + \varphi)}$$ Rov. vých.

$$\boxed{y = A_v \cdot \sin(\Omega t + \varphi)}$$