Introduction

[Introdakšn]

pre-term předtermín
[pri:term]

Physics study matter – in time / in space → properties (mass, volume); behaviour = motion, gravitation force, el –||–, mag –||–

Matter – substance (later) – body / particles – basic (electron, proton, neutron); composite (atoms, molecules)
– field

Clasic physics

until to the end of 19th century $v \ll c = 3 \cdot 10^8 \frac{m}{s}$

- mechanics
- acoustic
- wave
- optics
- Thermodynamics
- El + mag. field

Modern physics

20th century $v \doteq c$

Theorie of relativity

Quantum mechanics

Description – quality → units
kvantitativní
– quality – quantities – scalar – mag. m, A, V
kvalitativní — veličiny — vector – magnitude, direction
velocity $\vec{v}$
$\vec{F}$

Basic quantities + units (7)

SI (System International)

units

1. ~~length~~ length [lenght] l — [m]
2. mass [maes] m — [kg] [kilogram]
3. time t — [s]
4. el. current I [ai] — [A]
5. illumination I svítivost — [cd] candel
6. temperature ~~difference~~ T — [K] kelvin
7. ~~material~~ substance quantity n látkové množství — [mol] mol

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Derived [divaird] odvozené – quantities, units
Odvozené

$\frac{s}{t} = v \qquad \frac{m}{s} = \left[\frac{m}{s}\right] \qquad \frac{v}{t} = a \left[\frac{m}{s^2}\right]$

Secondary (2) vedlejší
minute, day, hour
liter, ~~ton~~ tonne
degree, celsius degree
electron volt

Supplementary Doplňkové
(2) rad – radian, steradian

Units partial and multiple
[paršl] [maltipl]

10^{-3}	m	mili	10^3	k	kilo
10^{-6}	μ	micro [maikrou]	10^6	M	mega
10^{-9}	n	nano [nanou]	10^9	G	giga
10^{-12}	p	pico [pikou]	10^{12}	T	terra

Mechanics [mekaniks]

1. Kinematics of particle
[kchinematiks]

Particle – has all attributes but not size
– mass, $\vec{v}$, $\vec{a}$
– only one point ⇒ cannot rotate
⇒ only translational movement
[transeišent]

Everything will be much easier

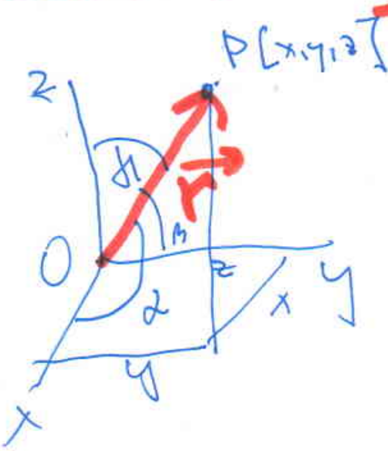
~~Position, trajectory~~

Kinematics – describe [diskraib] – motion < trajectory, velocity, acceleration

Dynamics – study reason of motion
reason is force < gravitational, el., mg

Position, trajectory

cartesian system framework $x \perp y \perp z$ are perpendicular



O .. origin, initial point, starting p.

x, y, z ; $v_x\, v_y\, v_z$ > coordinates

r .. [a:] position vector

$\vec{i}, \vec{j}, \vec{k}$ unit vectors / direction of axis \ ~~units~~ ~~states~~ scale
[ai, džej, kei]

$|\vec{r}|$ magnitude the square root of

$$|\vec{r}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

$\cos\alpha = \frac{v_x}{|\vec{r}|}$, $\cos\beta = \frac{v_y}{|\vec{r}|}$, $\cos\gamma = \frac{v_z}{|\vec{r}|}$

[kousaju]

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quantities < vector ($\vec{r}, \vec{a}, \vec{F}$) < magnitude, direction

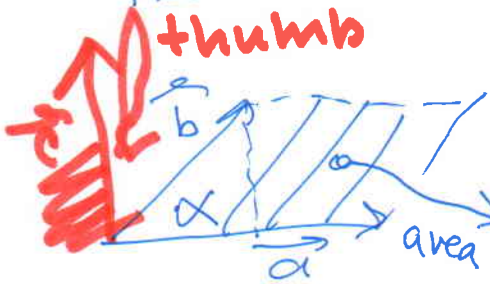
scalar – only magnitude

m, V, A

vector product
Vector product

$\vec{c} = \vec{a} \times \vec{b}$

The cross product



$\vec{c} = \vec{a} \times \vec{b}$

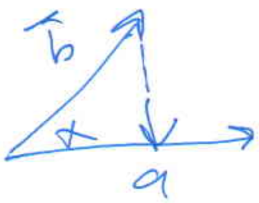
$|\vec{c}| = |\vec{a}| \cdot |\vec{b}| \cdot \sin\alpha$

Right-hand-rule

finger shows rotation from vector $\vec{a}$ to vector $\vec{b}$

Thumb shows direction of the vector c

Dot product
Scalar product

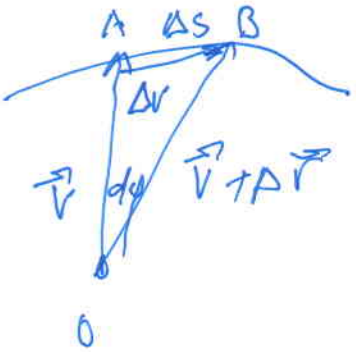


$c = \vec{a} \cdot \vec{b}$

$A = \vec{F} \cdot d\vec{r}$

$|c| = |\vec{a}| \cdot |\vec{b}| \cdot \cos\alpha$

2. Instantaneous velocity
[instantaneous]



average velocity = speed

$$v_s = \bar{v} = \frac{\Delta s}{\Delta t} \approx \frac{|\Delta \vec{r}|}{\Delta t} \quad \left[\frac{m}{s}\right]$$

Instantaneous velocity

$$A \to B \Rightarrow \Delta t \to 0$$

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$\boxed{\vec{v} = \frac{d\vec{r}}{dt}}$$

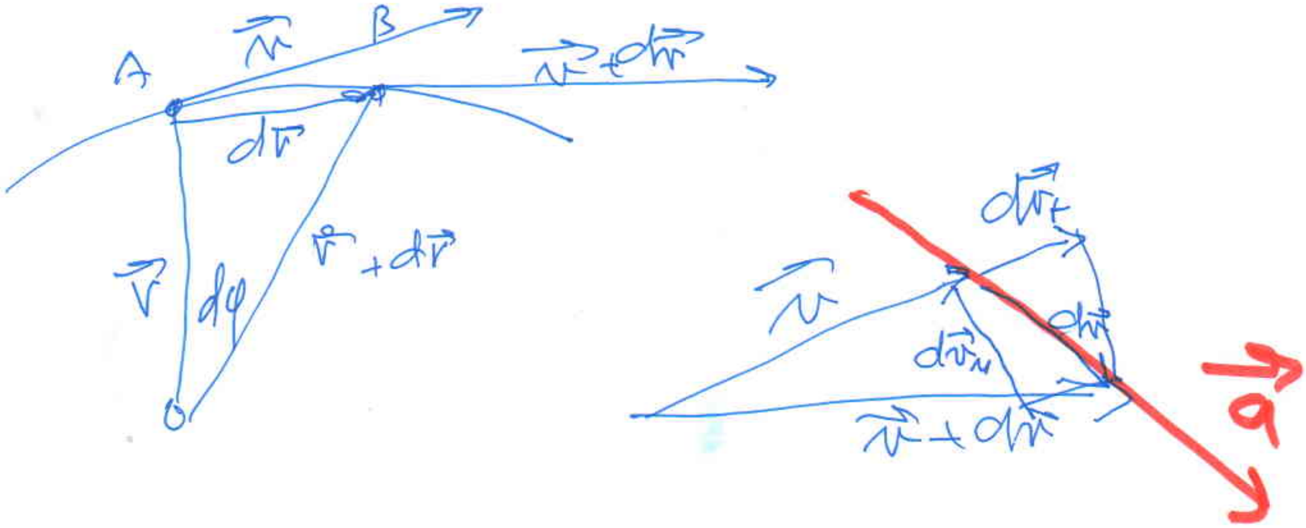
Instantaneous velocity is equal to first derivative of position vector with respect to t.

$$v_x = \frac{dr_x}{dt}, \quad v_y = \frac{dr_y}{dt}, \quad v_z = \frac{dr_z}{dt}$$

$$\boxed{\vec{v} = (v_x \vec{i} + v_y \vec{j} + v_z \vec{k}) \left[\frac{m}{s}\right]}$$

magnitude $|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$

3. Instantaneous acceleration



Average acceleration

$$a_s = \bar{a} = \frac{\Delta\vec{v}}{\Delta t} \simeq \frac{|\Delta v|}{\Delta t} = \frac{\text{change of velocity}}{\text{time taken for chang}} \approx \frac{|\Delta\vec{v}|}{\Delta t}$$

$\left[\frac{m}{s^2}\right]$

Instantaneous acceleration

$\Delta\varphi \to 0 \Rightarrow \Delta t \to 0$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

$$\boxed{\vec{a} = \frac{d\vec{v}}{dt}}$$

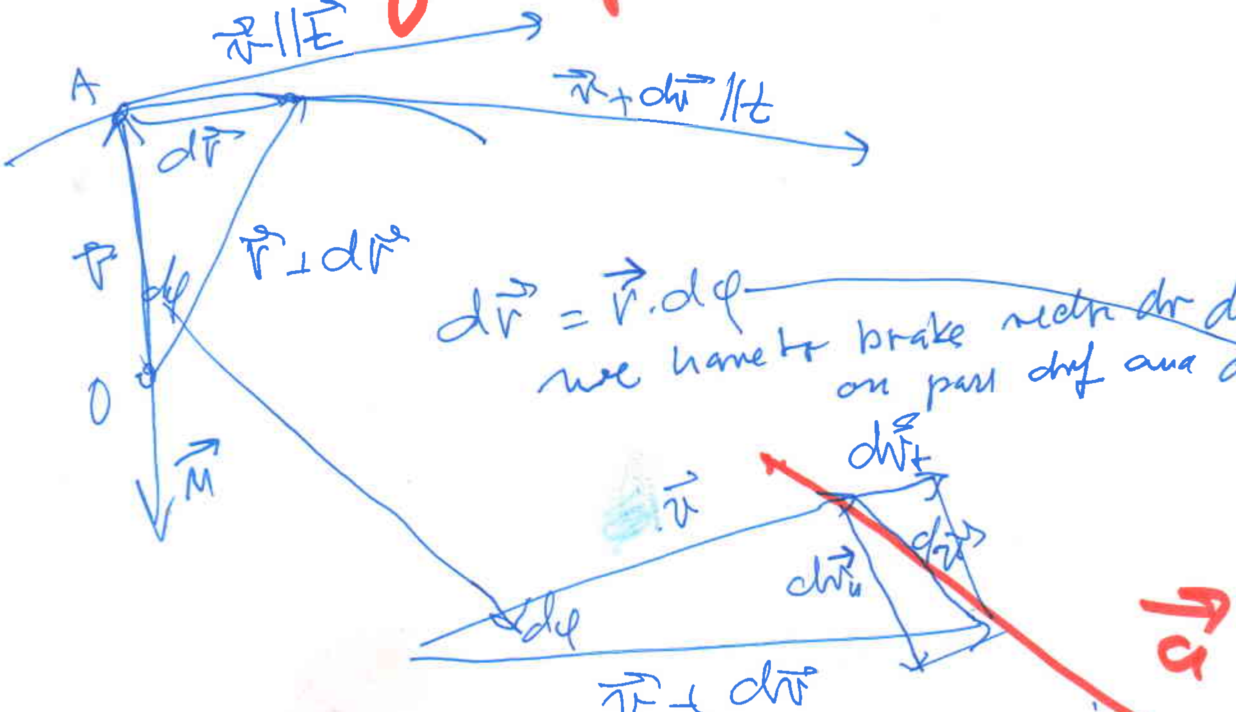
Instantaneous acceleration is equal to second derivative of position vector with respect to t.

$$a_x = \frac{dv_x}{dt}, \quad a_y = \frac{dv_y}{dt}, \quad a_z = \frac{dv_z}{dt}$$

$$\vec{a} = (a_x\vec{i} + a_y\vec{j} + a_z\vec{k}) \left[\frac{m}{s^2}\right]$$

magnitude $|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

4. Analysis of vector of acceleration



$d\vec r = \vec r \cdot d\varphi$

we have to brake vector $d\vec v$ down on part $d\vec v_t$ and $d\vec v_n$

if $d\varphi \to 0 \Rightarrow$ follows $d\vec v_t \rightrightarrows d\vec v$

$$\vec a_t = \frac{d\vec v_t}{dt} =$$

$$\boxed{\vec a_t = \frac{dv}{dt}}$$

$$\vec a_n = \frac{d\vec v_n}{dt} = \frac{\vec v \cdot d\varphi}{dt} = \frac{\vec v \cdot d\vec r}{dt \ \vec r} = \frac{v^2}{\vec r}$$

$$\boxed{\vec a_n = \frac{v^2}{r}}$$

$$\vec a = \vec a_t + \vec a_n$$

$$|\vec a| = \sqrt{a_t^2 + a_n^2}$$

5. General motion

$\vec{a} = 0$ — uniform motion (straight line)

$\vec{a} = const \neq 0$ motion with constant acceleration
is not equal to zero

$\vec{a} = \vec{a}(t)$ motion with general acceleration

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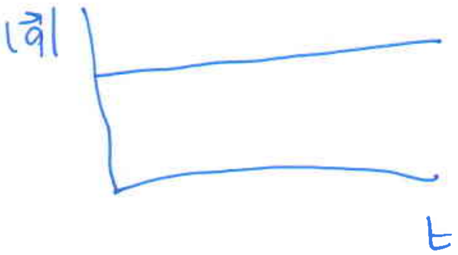
1) $\boxed{\vec{a} = constant \neq 0}$



$$\vec{a} = \frac{d\vec{v}}{dt}$$

Separate variable

$$d\vec{v} = \vec{a}\,dt$$

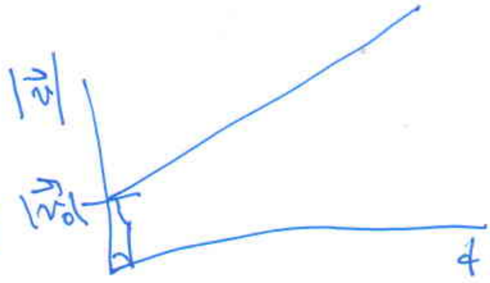
make integral

$$\int d\vec{v} = \int \vec{a}\,dt$$

$$\vec{v} + c_1 = \vec{a}t + c_2$$

$$\vec{v} = \vec{a}t + \underbrace{(c_2 - c_1)}_{\vec{v}_0}$$

$$\boxed{\vec{v} = \vec{a}t + \vec{v}_0}$$

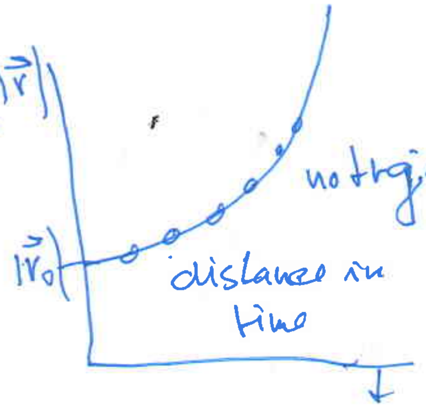


v_0 from edge condition
[edž]
($t = 0 \Rightarrow$ initial condition
[iniš])

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\int d\vec{r} = \int \vec{v}\,dt = \int (\vec{a}t + \vec{v}_0)\,dt$$

$$\boxed{\vec{r} = \frac{1}{2}at^2 + \vec{v}_0 \cdot t + \vec{r}_0}$$



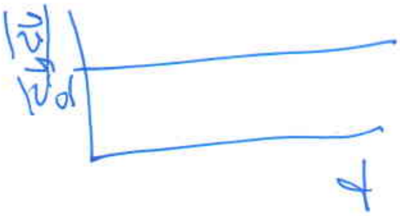
2, $\boxed{\vec{a} = 0}$ uniform motion

Rovnoměrný pohyb (nezrychlený)

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\int d\vec{v} = \int \vec{a}\, dt = \int 0\, dt$$

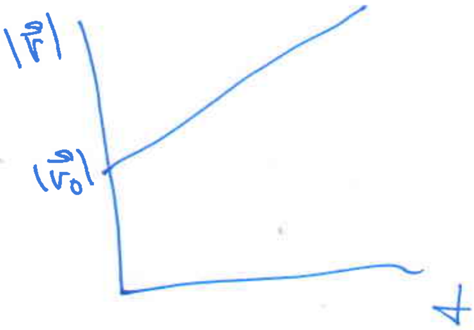
$$\boxed{\vec{v} = \vec{v}_0}$$



$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\int d\vec{r} = \int \vec{v}\, dt = \int \vec{v}_0\, dt$$

$$\boxed{\vec{r} = \vec{v}_0 \cdot t + \vec{r}_0}$$



Straight line motion

[streit lain moušn]

$$\vec{a} = (a_x, 0, 0)$$

motion with constant acceleration

$$a_x = \frac{dv_x}{dt}$$

$$\int dv_x = \int a_x dt$$

$$\boxed{v_x = a_x \cdot t + v_{x0}}$$

$$v_x = \frac{dr_x}{dt}$$

$$\int dr_x = \int v_x dt = \int (a_x t + v_{x0}) dt$$

$$\boxed{r_x = \frac{1}{2} a_x \cdot t^2 + v_{x0} \cdot t + r_{x0}}$$

$$\boxed{a_x = 0}$$

uniform straight motion

$$a_x = \frac{dv_x}{dt}$$

$$\int dv_x = \int 0 \, dt$$

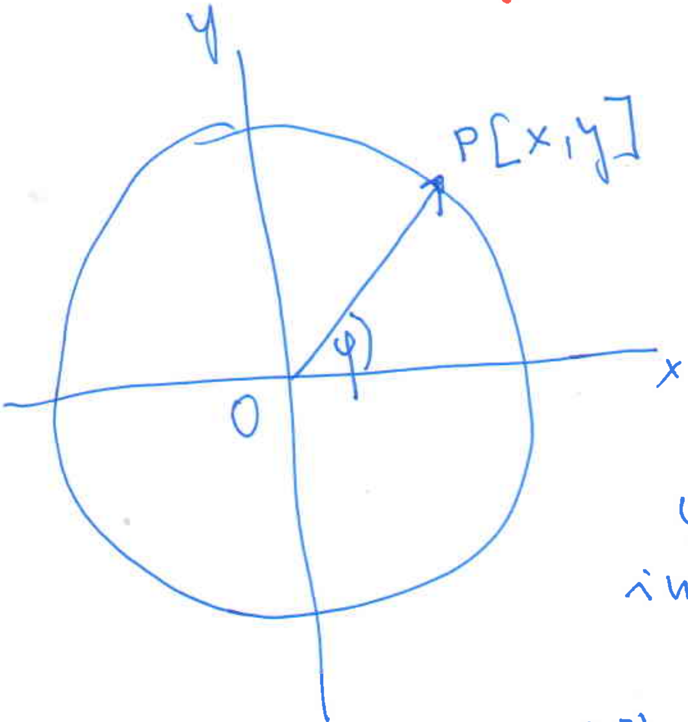
$$\boxed{v_x = v_{x0}}$$

$$v_x = \frac{dr_x}{dt}$$

$$\int dr_x = \int v_x dt = \int v_{x0} dt$$

$$\boxed{r_x = v_{x0} \cdot t + r_{x0}}$$

6. Uniform circular motion



$\vec{r}$ ... $\vec{\varphi}$ [1] angular angle [engel]
$\vec{v}$ ... $\vec{\omega}$ angular velocity [s^{-1}]
$\vec{a}$ ... $\vec{\varepsilon}$ [s^{-2}] ang. acceleration

$\varphi = \omega \cdot t$
increases linearly with time

$$x = |\vec{r}| \cdot \cos\varphi = |\vec{r}| \cdot \cos(\omega t)$$
$$y = |\vec{r}| \cdot \sin\varphi = |\vec{r}| \sin(\omega t)$$
$$\boxed{\vec{r} = |\vec{r}| \cdot \cos(\omega t)\,\vec{i} + |\vec{r}| \cdot \sin(\omega t)\,\vec{j}}$$
$$\vec{v} = \frac{d\vec{r}}{dt} = -\omega\,|\vec{r}| \cdot \sin(\omega t)\,\vec{i} + \omega\,|\vec{r}| \cdot \cos(\omega t)\,\vec{j}$$
$$\vec{a} = \frac{d\vec{v}}{dt} = -\omega^2 |\vec{r}| \cdot \cos(\omega t)\,\vec{i} - \omega^2 |\vec{r}| \cdot \sin(\omega t)\,\vec{j}$$
$$\boxed{\vec{a} = -\omega^2 \vec{r}}$$

centripetal acceleration
[sentripete]
Directed towards the center of curvature.

$\vec{v}$ ... peripheral velocity
[periferel]

We know $|\vec{a}_n| = \left|\frac{v^2}{r}\right| = \left|-\omega^2 \vec{r}\right|$ scalar equation

$$\Rightarrow \boxed{v = r \cdot \omega}$$

$$\vec{a}_t = \frac{d\vec{v}}{dt} = \frac{d(r \cdot \omega)}{dt} = 0$$ r, ω are constants

[tandžiwl]

T - [s] ... period

$\frac{1}{T} = f \; [s^{-1} = Hz]$ frequency [frikwensi]

$\varphi = \omega t$

$2\pi = \omega \cdot T$

$\Rightarrow T = \frac{2\pi}{\omega}$

Length of trajectory

$$\frac{dx}{dt} = -\omega |\vec{r}| \cdot \cancel{\cos}\ \sin(\omega t)$$

$$dx = -\omega |\vec{r}| \cdot \sin(\omega t) dt$$

$$\frac{dy}{dt} = \omega |\vec{r}| \cdot \cos(\omega t)$$

$$dy = \omega |\vec{r}| \cdot \cos(\omega t) \cdot dt$$

$$\int ds = \int_0^t \sqrt{dx^2 + (dy)^2}$$

$$S = \int_0^t \sqrt{\omega^2 |\vec{r}|^2 \cdot \cos^2(\omega t) + \omega^2 |\vec{r}|^2 \sin^2(\omega t)}\, dt$$

$$S = \int_0^t \sqrt{\omega^2 |\vec{r}|^2 \cdot (\underbrace{\cos^2(\omega t) + \sin^2(\omega t)}_{1})}\, dt$$

$$S = \omega \cdot |\vec{r}| \cdot t$$

$$\boxed{S = r \cdot \varphi} \qquad \boxed{S_c = r \cdot 2\pi}$$

7. Circular motion with uniform angular acceleration

$$\boxed{\vec{\varepsilon} = \text{constant} \neq 0}$$

Only scalar equation

$$\varepsilon = \frac{d\omega}{dt}$$

$$\int d\omega = \int \varepsilon \, dt$$

$$\boxed{\omega = \varepsilon \cdot t + \omega_0}$$

$$\omega = \frac{d\varphi}{dt}$$

$$\int d\varphi = \int \omega \, dt = \int (\varepsilon t + \omega_0) \, dt$$

$$\boxed{\varphi = \frac{1}{2}\varepsilon t^2 + \omega_0 t + \varphi_0}$$

Peripheral velocity

$$v = r \cdot \omega = r \cdot \varepsilon \cdot t + r \cdot \omega_0$$

But $$a_t = \frac{dv}{dt} = \frac{d(r\varepsilon t + r \cdot \omega_0)}{dt} = r \cdot \varepsilon$$

$$a_t = r \cdot \varepsilon \quad \text{is not zero !!}$$

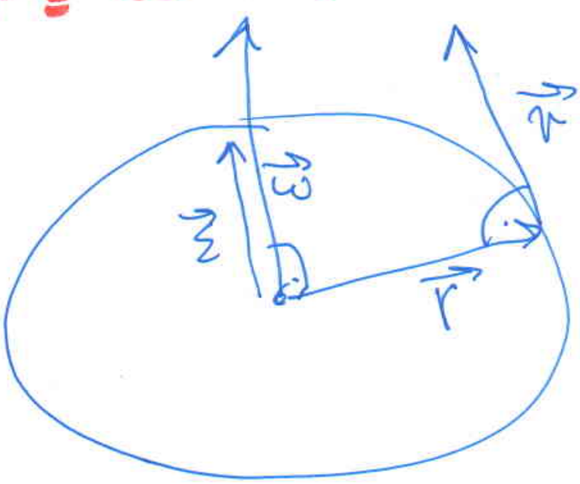
$$a_n = \frac{v^2}{r}$$

$$|\vec{a}| = \sqrt{a_t^2 + a_n^2}$$

Vector equations

$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$\vec{\varepsilon} = \frac{d\vec{\omega}}{dt}$$



Right-hand-rule

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d(\vec{\omega} \times \vec{r})}{dt} = \underbrace{\left(\frac{d\vec{\omega}}{dt} \times \vec{r}\right)}_{\vec{a}_t} + \underbrace{\left(\vec{\omega} \times \frac{d\vec{r}}{dt}\right)}_{\vec{a}_n}$$

$$\vec{a}_t = \vec{\varepsilon} \times \vec{r}$$

$$\vec{a}_n = \vec{\omega} \times \vec{v}$$