Hydromechanics

- hydrostatic
- hydrodynamics

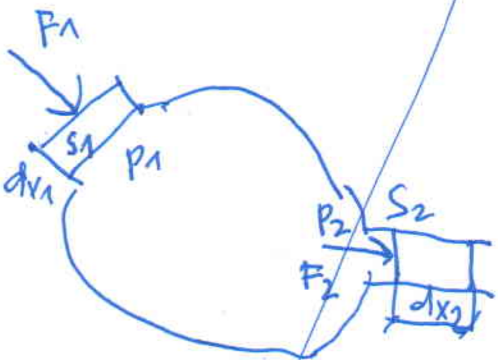
fluid
- liquid (kapalina)
- gas (plyn)

Pressure $p = \frac{dF}{dS}$ $\left[\frac{N}{m^2} = Pa\right]$

$\vec{F} = \int_{\vec{S}} p\, d\vec{S}$ [N]

33. Pascal's principle

External force



$$dW_1 = F_1 \cdot dx_1 = p_1 \cdot \underbrace{(S_1 \cdot dx_1)}_{dV_1} = p_1 dV_1$$

$$dW_2 = F_2 \cdot dx_2 = p_2 \cdot S_2\, dx_2 = p_2 dV_2$$

$$dW_1 = dW_2$$

$$p_1 \cancel{dV_1} = p_2 \cancel{dV_2}$$

incompressible [inkompresibl] nestlačitelný

$$\boxed{p_1 = p_2}$$

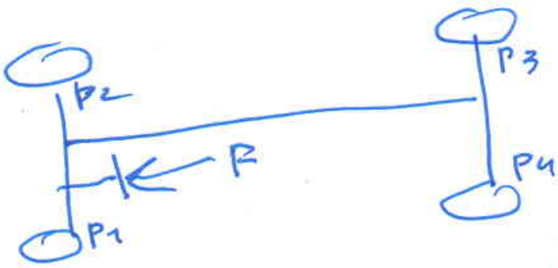
The pressure due to an external force is in the whole volume the same.

For gas is valid the same

Use – hydraulic press

$$\left.\begin{array}{l} p_1 = p_2 \\ \dfrac{F_1}{S_1} = \dfrac{F_2}{S_2} \end{array}\right\} \Rightarrow F_2 = F_1 \cdot \underbrace{\frac{S_2}{S_1}}_{\text{ratio}}$$

hydraulic brake system



$p_1 = p_2 = p_3 = p_4$

$S_1 = S_2 = S_3 = S_4$

$\Rightarrow F_1 = F_2 = F_3 = F_4$

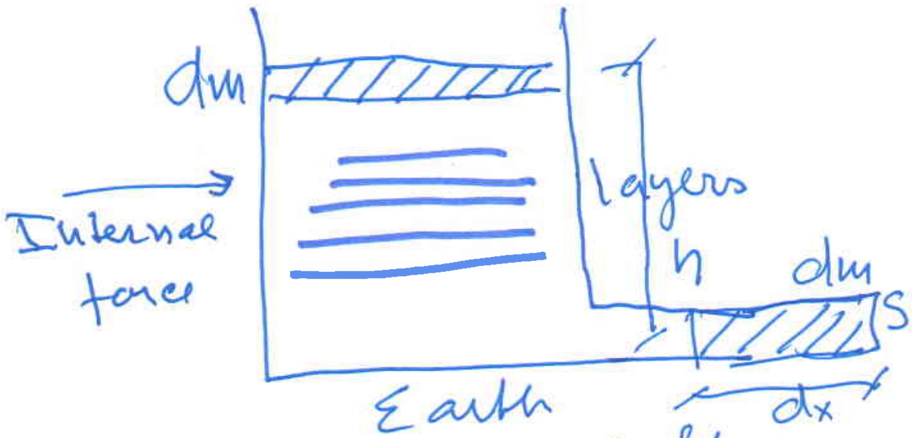
brake pads

$\Rightarrow$ The car doesn't skid

brzdové destičky

34. Hydrostatic pressure

Due to an internal forces (in gravitation field)



Gravitation field

Gravitation force on the layers

$$dmgh = dW = F \cdot dx = p\,\underbrace{(S \cdot dx)}_{dV}$$

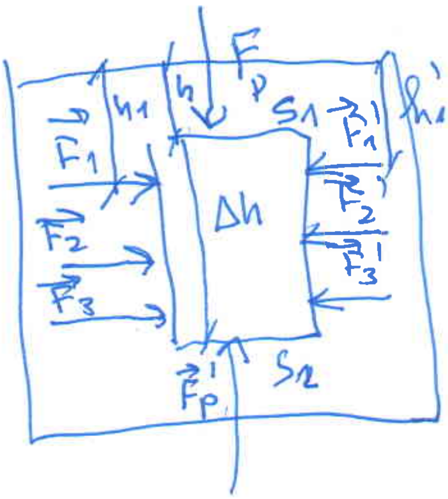
$$dmgh = p \cdot dV$$

$$\boxed{P_h = h\rho g}$$ Hydrostatic pressure

In liquid

In gas

35. Archimede's principle



$S_1 = S_2 = S$

$\left.\begin{array}{l} \vec{F}_1 = -\vec{F}_1' \\ h_1 = h_1' \\ \vec{F}_2 = \vec{F}_2' \\ \vec{F}_3 = \vec{F}_3' \end{array}\right\} \sum \vec{F} = 0$

$h_1 = h_1'$

F_{vz} Buoyant force [Bojent] [Buoyant] nadnášivá sila

$$F_{vz} = F_p - F_p' = pS_1 - p'S_2'$$

$$F_{vz} = h\rho g S - (h+\Delta h)\rho g S$$

$$F_{vz} = \cancel{h\rho g S} - \cancel{h\rho g S} - \overset{V}{\Delta h \cdot S}\,\rho g$$

$$\boxed{F_{vz} = -\rho g V}$$

Archimede's principle

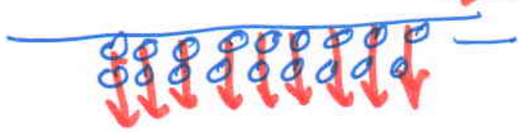
The buoyant force on a body has the same magnitude as the weight of the body push up and opposite direction as gravitation force.
Is valid for gas too

36. Surface tension

Povrchové napětí

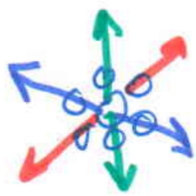
[serfis]

$$\Rightarrow \sigma = \frac{F}{l}\left[\frac{N}{m}\right]$$



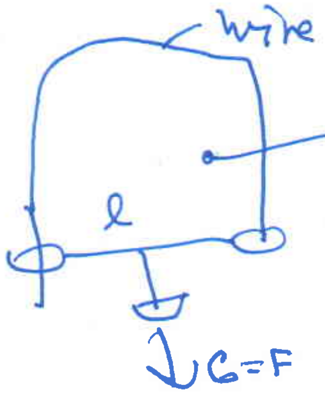
the last line

molecules in the last line are only under forces down



The internal molecules are in equilibrium

$$\sum F = 0$$



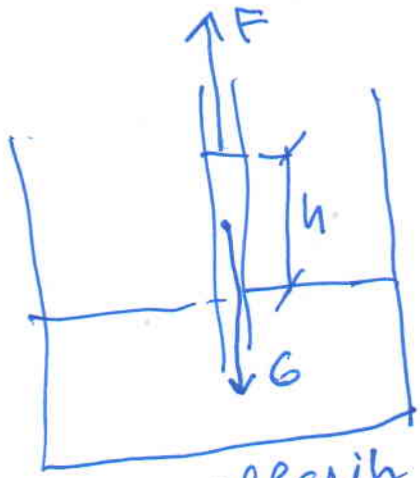
soapbubble
mýdlová bublina
(hollow) $\Rightarrow 2x$
dutý

$\downarrow G = F \quad \sigma = \frac{F}{l}\cdot\frac{1}{2}$

$$\sigma = \frac{F}{l}\cdot\frac{dr}{dr} = \frac{dW}{dS} = \frac{dE}{dS}\left[\frac{J}{m^2}\right]$$

areas' density of energy

Plošná hustota energie



capillarity

$$G = F$$

$$(\pi r^2 \cdot h)\cdot\rho\cdot g = \sigma\cdot l = \sigma\cdot 2\pi r$$

$$\Rightarrow \left\langle \begin{matrix}\sigma\\ r\end{matrix}\right.$$

37. equation of continuity

We use streamlines to describe — laminar / turbulent [terbjulent]
proudní čáry

laminar

turbulent

Constant flow
ustálené proudění — velocity, pressure, temperature — At a given location, doesn't change with time

Volume flow

$$\boxed{Q_V = \frac{dV}{dt} \left[\frac{m^3}{s}\right]}$$

$$Q_V = \frac{S \cdot dl}{dt} = S \cdot v$$

$$\boxed{Q_V = S \cdot v}$$

Mass flow

$$Q_m = \frac{dm}{dt} = \frac{\rho \cdot dV}{dt} = \rho \cdot S \cdot v$$

$$\boxed{Q_m = \rho \cdot S \cdot v \;[kg \cdot s^{-1}]}$$

1) Incompressible fluid (= liquid)

Law of conservation of mass

Nonuniformity tubing nerovnoměrné p.

medium doesn't even arise

medium doesn't even get lost

$$Q_{V1} = Q_{V2}$$

$$\boxed{S_1 \cdot v_1 = S_2 v_2}$$

Equation of continuity for incompressible fluid

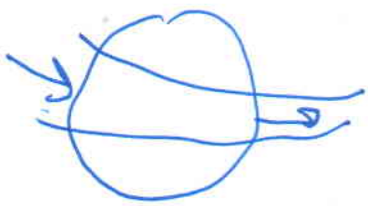
2) Compressible fluid (gas)

$$Q_{m1} = Q_{m2}$$

$$\boxed{\rho_1 S_1 v_1 = \rho_2 S_2 v_2}$$

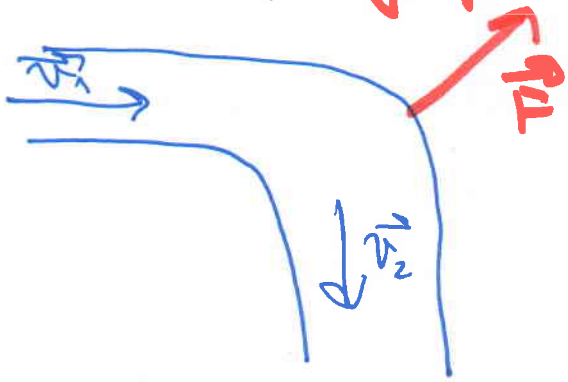
Equation of continuity for compressible fluid

3) generally



$$\oint \rho v \, dS = 0$$

38. Principle of fluid momentum



Impulse of force

$$\vec{I} = \vec{F}\cdot\Delta t = \Delta\vec{p} = \Delta m\left(\vec{v}_2 - \vec{v}_1\right) \Big/ \Delta t$$

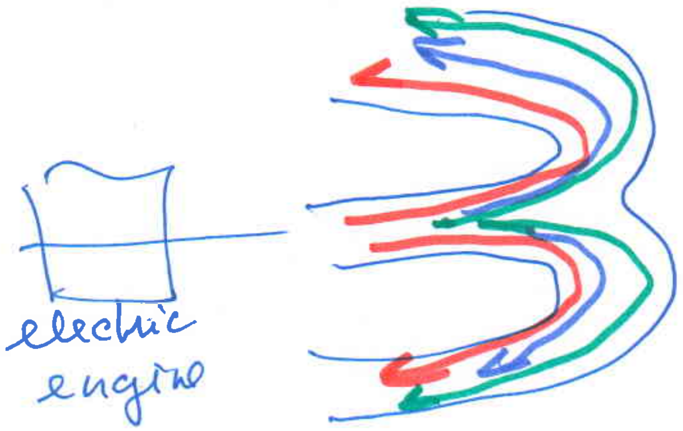
$$\vec{F} = \frac{\Delta m}{\Delta t}\left(\vec{v}_2 - \vec{v}_1\right)$$

$$\boxed{\vec{F} = Q_m\cdot\left(\vec{v}_2 - \vec{v}_1\right)}$$

Vector diference

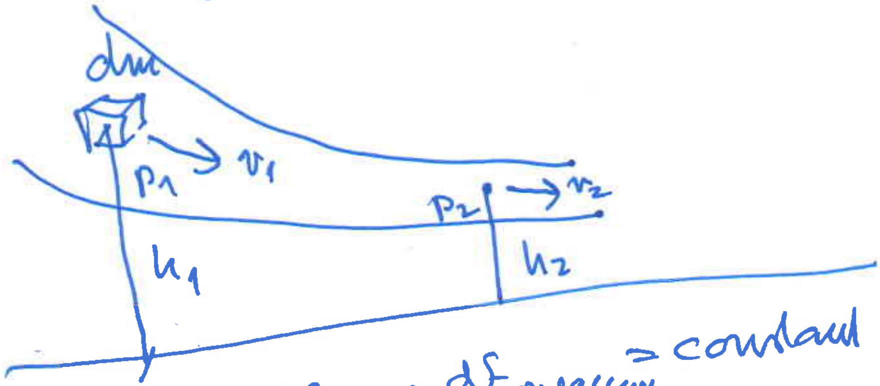
Fluid action by force on a body (tube), and depends on mass flow Q_m and change of velocity vectors $(\vec{v}_2 - \vec{v}_1)$

Pelton turbine



39. Bernoulli's equation

Law of conservation of Energy



$dE_k + dE_p + dE_{preassure} = constant$

kinetic potential preassure

$\frac{1}{2} dm \cdot v^2 + dm\,g\,h + p\,dV = constant \,/ dV$

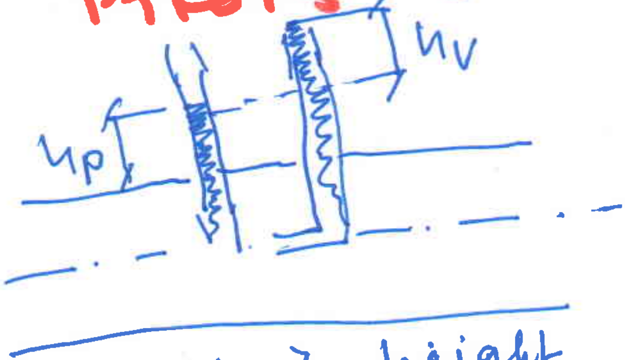
$$\boxed{\frac{1}{2}\rho v^2 + \rho g h + p = constant}$$

Bernoulli's eq.

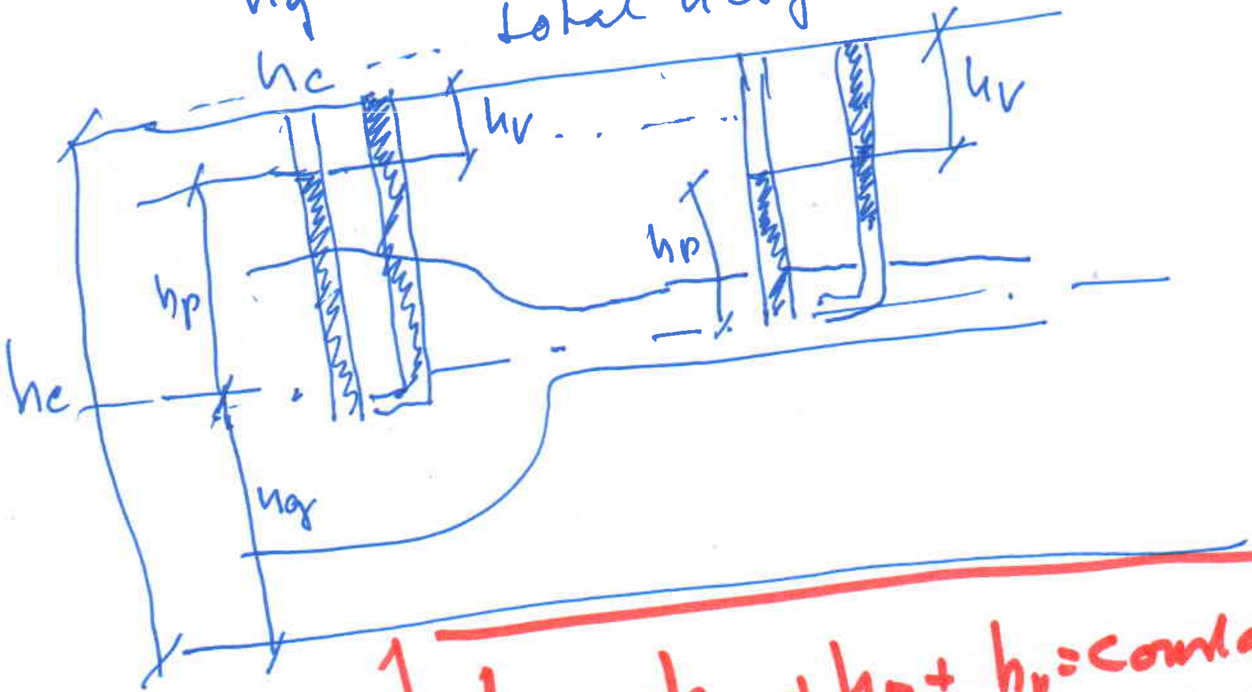
Or

$$\frac{1}{2}\rho v_1^2 + \rho g h_1 + p_1 = \frac{1}{2}\rho v_2^2 + \rho g h_2 + p_2$$

Pitot's tube



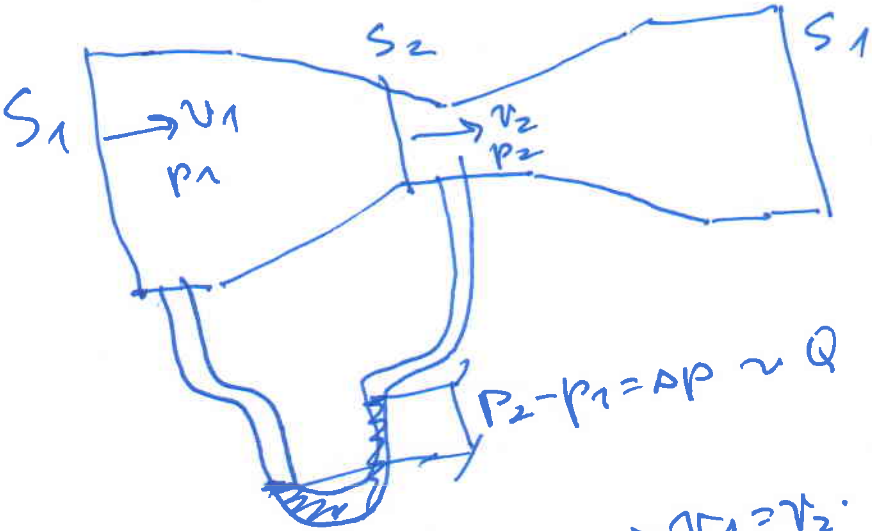
h_v ... velocity height
h_p ... preassure height
h_g .... conventional height
h_c .... total height



$$\boxed{h_c = h_g + h_p + h_v = \text{constant}}$$

Bernoulli's equation
Height equation

Venturi's tube



$p_2 - p_1 = \Delta p \sim Q$

$$S_1 v_1 = S_2 v_2 \rightarrow v_1 = v_2 \cdot \frac{S_2}{S_1}$$

$$\frac{1}{2}\rho v_1^2 + p_1 + \cancel{h_1 \rho g} = \frac{1}{2}\rho v_2^2 + p_2 + \cancel{h_2 \rho g}$$

$h_1 = h_2$

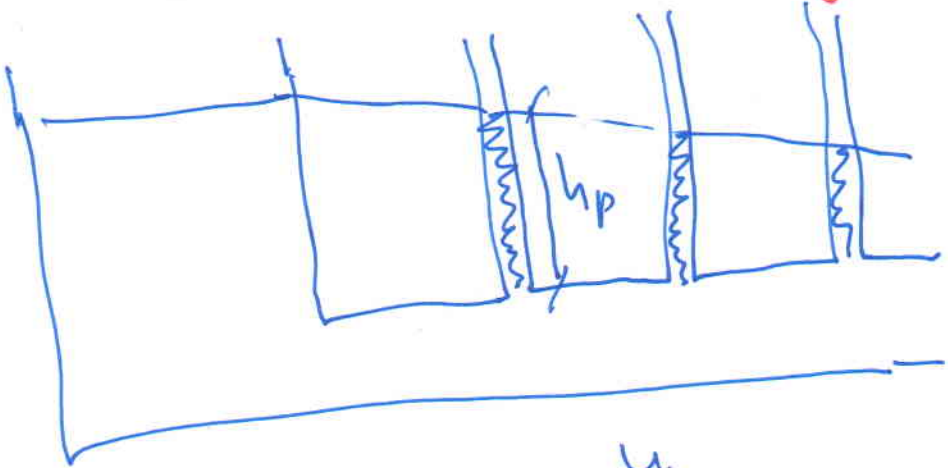
$$\frac{1}{2}\rho\, v_2^2 \frac{S_2^2}{S_1^2} + p_1 = \frac{1}{2}\rho v_2^2 + p_2$$

$$\frac{1}{2}\rho\left(\frac{S_2^2}{S_1^2} - 1\right) \cdot v_2^2 = p_2 - p_1$$

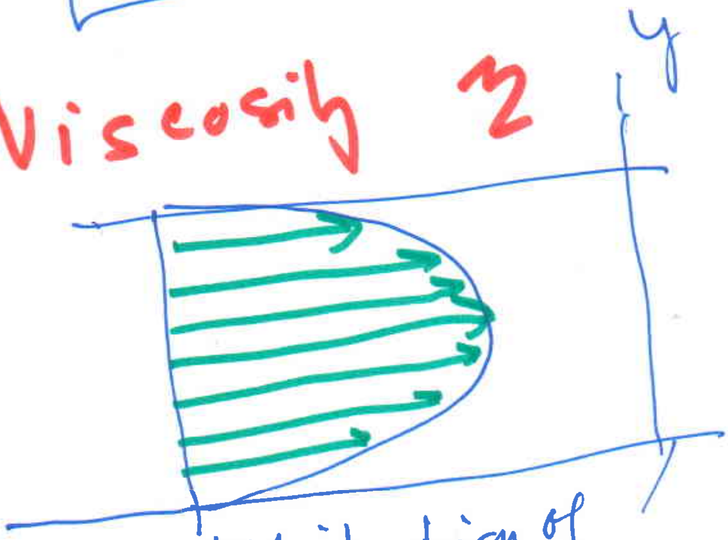
$$v_2 = \sqrt{\frac{(p_2 - p_1)\cdot 2}{\rho\left(1 - \frac{S_2^2}{S_1^2}\right)}}$$

$$\boxed{Q_V = S_2 \cdot v_2} \rightarrow \text{Quantity of volume flow}$$

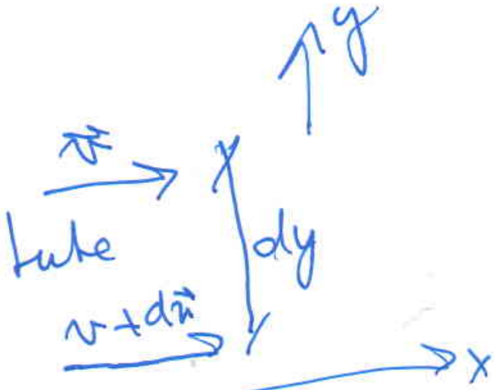
Motion of real fluid (liquid)



Viscosity η



distribution of velocity



The velocity gradient

due to friction among layers

The velocity at the wall is → zero

τ .. tangential stress

$$\tau = \frac{F}{S} \left[\frac{N}{m^2}\right]$$

$$\tau = \eta \cdot \frac{dv}{dy}$$

$$\boxed{\eta = \tau \cdot \frac{1}{\frac{dv}{dy}} \; [Pa \cdot s]}$$

dynamic viscosity

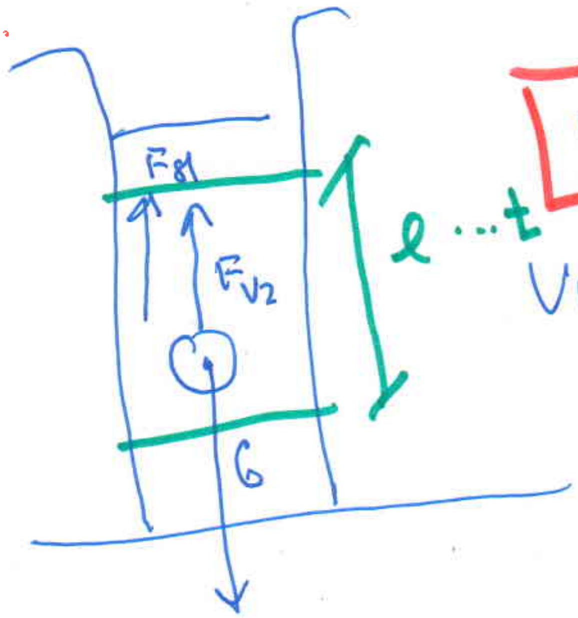
η is the tangencial force per unit area of one layer fluid which resists the motion of one layer over another.

Body shape O ⊃ I ⊂ – hemisphere

Stokes formula for sphere $V = \frac{4}{3}\pi r^3$

$$\boxed{F_{f1} = 6\pi\eta \cdot v \cdot r}$$

Stoke's viscometer



$$\boxed{F_{v2} + F_{f1} = G} \Rightarrow v = \text{constant}$$

$$V\rho g + 6\pi\eta \cdot v \cdot r = \frac{4}{3}\pi r^3 \rho g$$