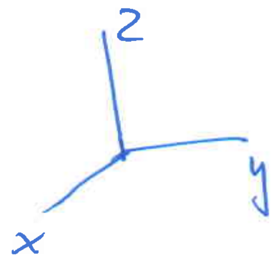
20. Mass and momentum of system of particles, external and internal forces

i – degree of freedom
[digri:]

One particle has three degree of freedom $i = 3$

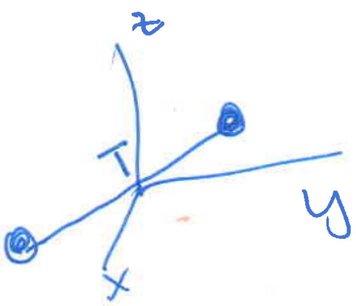


On plane  $i = 2$

On trajectory $i = 1$

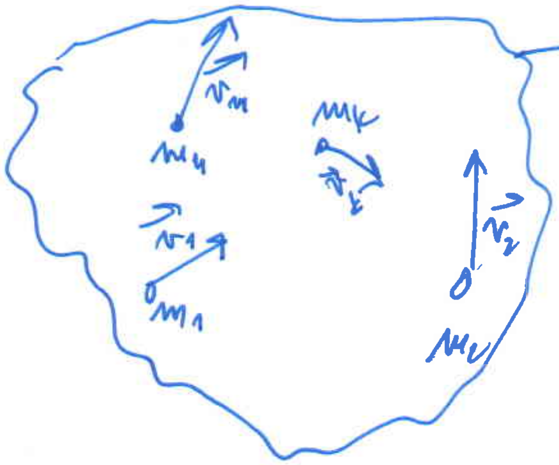
Two particles with binding

ℓ .... binding



Center 3
α, β $\frac{2}{5}$ degree of freedom

In general $i = 3 \cdot n - \ell$
$i = 3 \cdot 2 - 1 = 5$



Define what belongs there

Mass of system of particles

$$m = m_1 + m_2 + \ldots + m_n$$

add up

$$\boxed{m = \sum_{k=1}^{n} m_k}$$

Momentum of system of particles

$$\vec{p} = \vec{p}_1 + \vec{p}_2 + \ldots + \vec{p}_n$$

$$\boxed{\vec{p} = \sum_{k=1}^{n} \vec{p}_k}$$

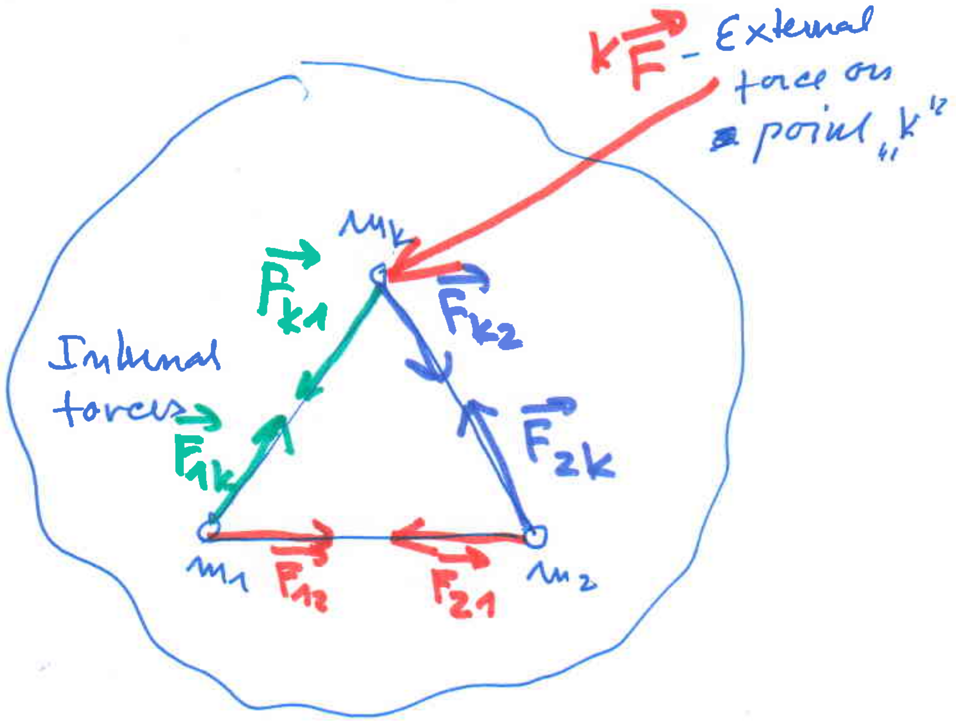
External and internal forces of SofP.

$\vec{F}_{12} = -\vec{F}_{21}$

$\vec{F}_{1k} = -\vec{F}_{k1}$

$\vec{F}_{2k} = -\vec{F}_{k2}$

${}^{k}\vec{F}$ - external force



Total force acting on particle „k"

$$\boxed{\vec{F}_k = \sum_{\substack{j=1 \\ j\neq k}}^{n} \vec{F}_{kj} + {}^{k}\vec{F}}$$

Resulting force acting on whole system of partic-

$$\boxed{\vec{F} = \sum_{k=1}^{n} \vec{F}_k = \underbrace{\sum_{k=1}^{n}\sum_{\substack{j=1 \\ j\neq k}}^{n} \vec{F}_{kj}}_{\text{Internal forces}} + \overbrace{\sum_{k=1}^{n} {}^{k}\vec{F}}^{\text{External forces}}}$$

Example $n=3$

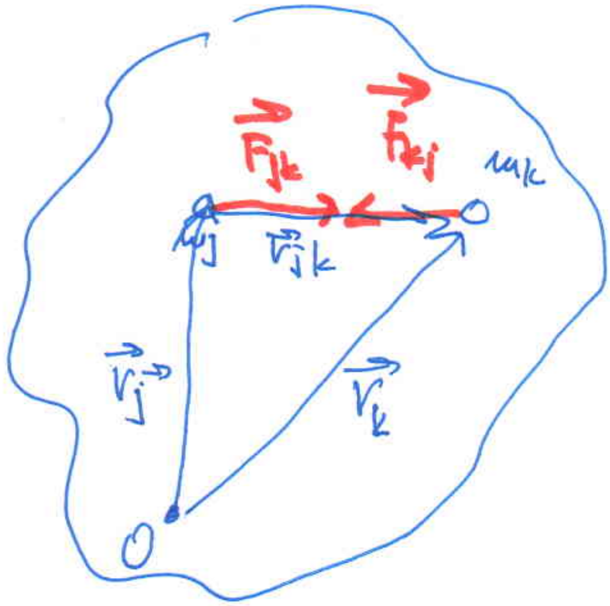
$$\sum_{k=1}^{3}\sum_{\substack{j=1 \\ j\neq k}}^{3} \vec{F}_{kj} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{21} + \vec{F}_{23} + \vec{F}_{31} + \vec{F}_{32} = 0$$

$$\boxed{\vec{F} = \sum_{k=1}^{n} {}^{k}\vec{F}}$$

The resulting force acting on SofP is equal to resulting external force acting on SofP.

21. Moment of internal forces

$$\vec{F}_{jk} = -\vec{F}_{kj}$$



$$\vec{M}_{kj} = \vec{r}_k \times \vec{F}_{kj}$$

$$\vec{M}_{jk} = \vec{r}_j \times \vec{F}_{jk}$$

Resulting moment

$$\vec{M} = \vec{M}_{kj} + \vec{M}_{jk} \qquad \vec{F}_{jk} = -\vec{F}_{kj}$$

$$\vec{M} = (\vec{r}_k \times \vec{F}_{kj}) + (\vec{r}_j \times \vec{F}_{jk})$$

$$\vec{M} = \underbrace{(\vec{r}_k - \vec{r}_j)}_{\vec{r}_{kj} \parallel \vec{F}_{kj}} \times \vec{F}_{kj} = 0$$

$$\boxed{\sum_{k=1}^{n} \sum_{j=1}^{n} \vec{M}_{jk} = 0}$$

Resulting moment of all internal forces of S of P is zero. [zirou]

Internal forces cannot bring motion of whole system of particles.

It can do only external forces.

22. Center of mass of system of particles.

It is necessary to calculate n - equations

It can be easier → Center of mass

1) There is total mass $m = \sum_{k=1}^{n} m_k$

2) On there point act resulting external force $\vec{F} = \sum_{k=1}^{n} {}^{k}\vec{F}$

3) Momentum of center of mass is equal to whole momentum of system of particles

m^*, $\vec{r}^*$, $\vec{v}^*$, $\vec{a}^*$ $*$ ..... mass center

$$m\vec{v}^* = \sum_{k=1}^{n} m_k \vec{v}_k$$

$$\downarrow \quad \frac{d(m\vec{r}^*)}{dt} = \frac{d(\sum m_k \vec{r}_k)}{dt}$$

after separation and integracion

$$\boxed{\vec{r}^* = \frac{\sum_{k=1}^{n} m_k \vec{r}_k}{\sum_{k=1}^{n} m_k}}$$

$$r_x^* = \frac{\sum m_k r_{xk}}{\sum_{k=1}^{n} m_k}$$

r_y

r_z

$$\vec{r}^* = (r_x\vec{i} + r_y\vec{j} + r_z\vec{k}) \left[m \right]$$

Earth   center of mass = center of gravity
těžiště

23. First impulse principle

For „k"- particle

$$\frac{d\vec{p}_k}{dt} = \vec{F}_k \qquad \vec{p}_k = m_k \vec{v}_k$$

$$\vec{F}_k = \underbrace{\sum_{\substack{j=1 \\ j\neq k}}^{n} \vec{F}_{kj}}_{\text{Internal f.}} + \underbrace{{}^{k}\vec{F}}_{\text{External f.}}$$

For all particles

$$\sum_{k=1}^{n} \frac{d\vec{p}_k}{dt} = \underbrace{\sum_{k=1}^{n}\sum_{\substack{j=1 \\ j\neq k}}^{n} \vec{F}_{kj}}_{\substack{\text{Resultant of} \\ \text{internal forces}} \; = \; 0} + \underbrace{\sum_{k=1}^{n} {}^{k}\vec{F}}_{\substack{\text{Resultant of} \\ \text{external forces}}}$$

$$\boxed{\frac{d\vec{p}}{dt} = \vec{F}}$$

The first impulse principle.

The change of resulting momentum of a system of particles is equal to resulting external forces.

If $\boxed{\vec{F} = 0}$ (isolated system)

Whole momentum $\vec{p}$ is constant

$$\boxed{\vec{p} = \text{constant}}$$

24. Second impulse principle

For particle „k"

$$\frac{d\vec{b}_k}{dt} = \vec{M}_k \qquad \vec{b}_k = \vec{r}_k \times (m_k \vec{v}_k)$$

$$\vec{M}_k = \vec{r}_k \times \vec{F}_k = \vec{r}_k \times \Big(\sum_{\substack{j=1 \\ j \neq k}}^{n} \vec{F}_{kj} + {}^{k}\vec{F}\Big)$$

$$M_k = \underbrace{\Big(\vec{r}_k \times \sum_{\substack{j=1 \\ j \neq k}}^{n} \vec{F}_{kj}\Big)}_{\text{Internal force} \;\Rightarrow 0} + \Big(\vec{r}_k \times {}^{k}\vec{F}\Big)$$

$$\frac{d\vec{b}}{dt} = \vec{M}$$ The second impulse principle.

The time change of resultant of moment of momentum of system of particles is equal to resultant moment of external forces

If $\boxed{\vec{M} = 0} \Rightarrow \boxed{\vec{b} = \text{constant}}$

25. Total rigid body, forces acting on the body

The distances between the points do not change among

Density $\rho = \frac{dm}{dV}$

Mass $m = \int_V \rho \, dV$

Position of the mass point

$$\boxed{\vec{r}^* = \frac{\int \vec{r} \, dm}{\int dm}}$$

$$x^* = \frac{\int x \, dm}{\int dm}$$

$$y^* = \frac{\int y \, dm}{\int dm}$$

$$z^*$$

Degree of freedom

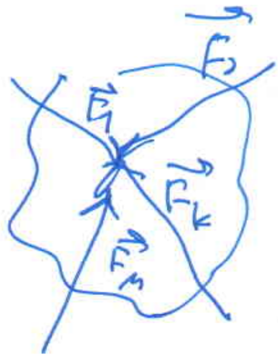
The position is determined by three points that do not lie in a straight line

$i = 3n - l = 3 \cdot 3 - 3 = 6$ degree of freedom.

$x, y, z, \alpha, \beta, \gamma$!

Resultant force

1) Forces in one point $\vec{F} = \sum_{k=1}^{n} \vec{F}_k$

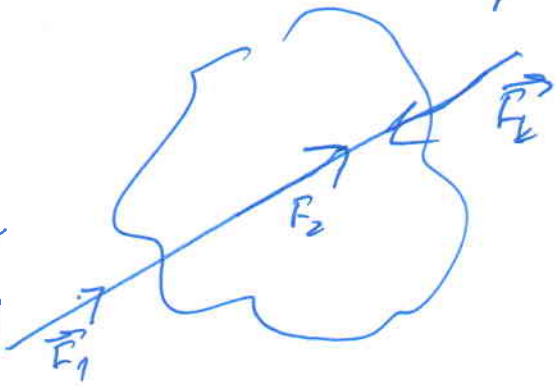


2) Straight line forces

$$\vec{F} = \sum_{k=1}^{n} F_k$$

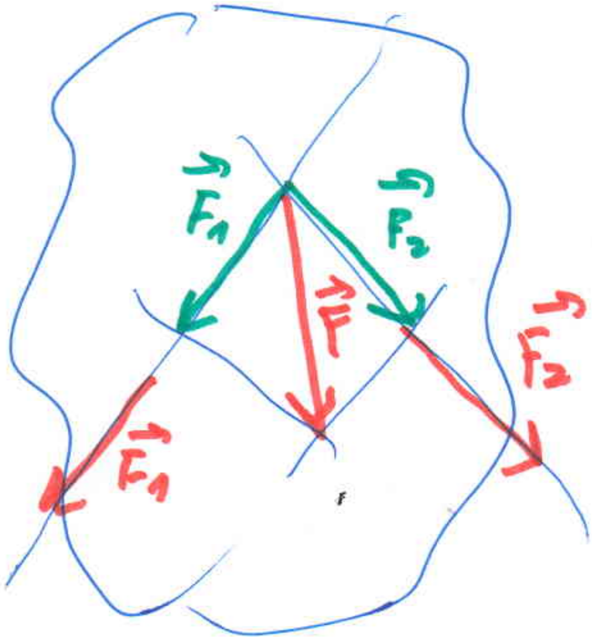
Resultant is a gliding vector [glaiding]

klouzavý vektor



3) Intersecting forces in the same plane

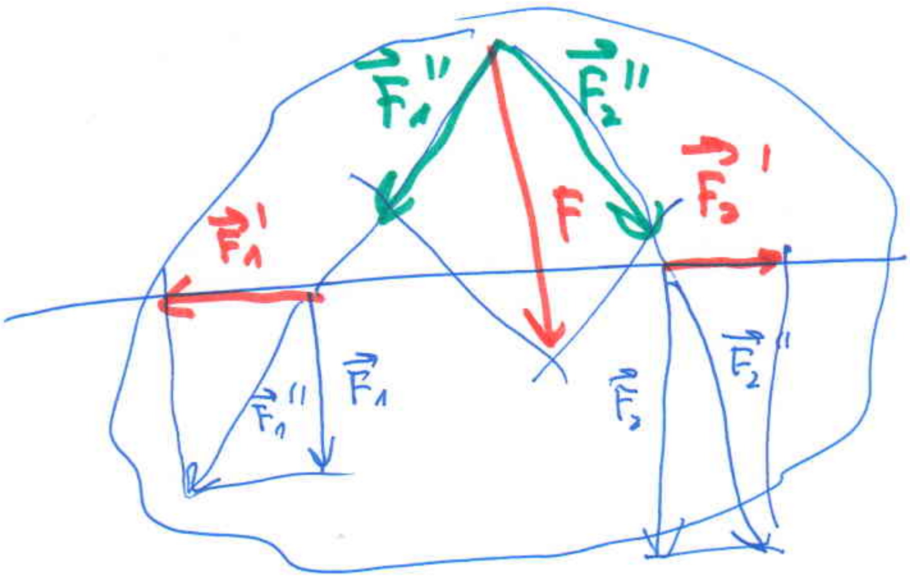
$$\vec{F} = \vec{F}_1 + \vec{F}_2$$



4, Parallel fores in the same plane

$$\vec{F}_1' = -\vec{F}_2'$$

$$\vec{F} = \vec{F}_1 + \vec{F}_2$$

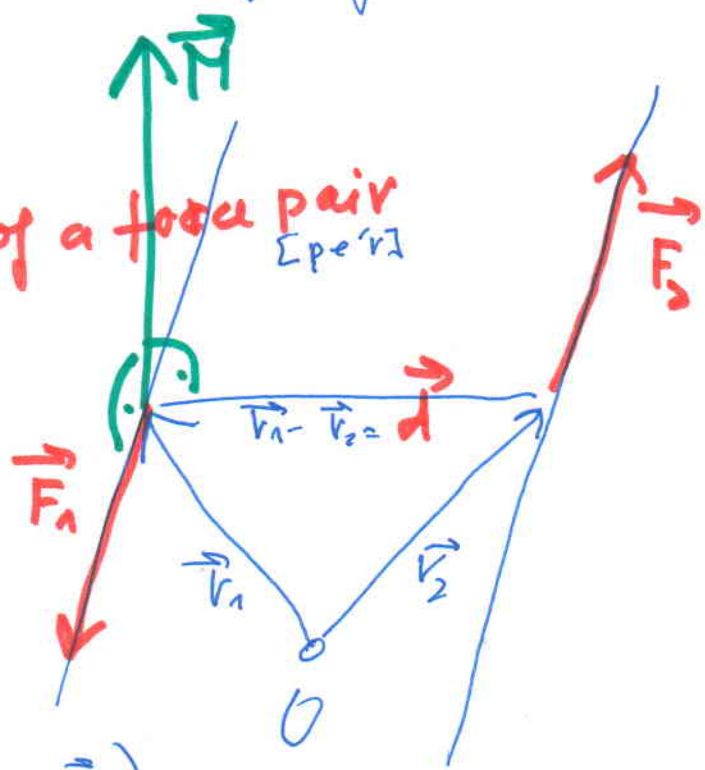


26. Couples forces

Two forces with the same magnitude opposite direction, not lying in the same line.

$\vec{F}_1 = -\vec{F}_2$

$\vec{d} = \vec{r}_1 - \vec{r}_2$ arm of a force pair [pe'r]



$\vec{M} = (\vec{r}_1 \times \vec{F}_1) + (\vec{r}_2 \times \vec{F}_2)$

$\vec{M} = (\vec{r}_1 - \vec{r}_2) \times \vec{F}_1 = \vec{d} \times \vec{F}_1$

$$\boxed{\vec{M} = \vec{d} \times \vec{F}}$$

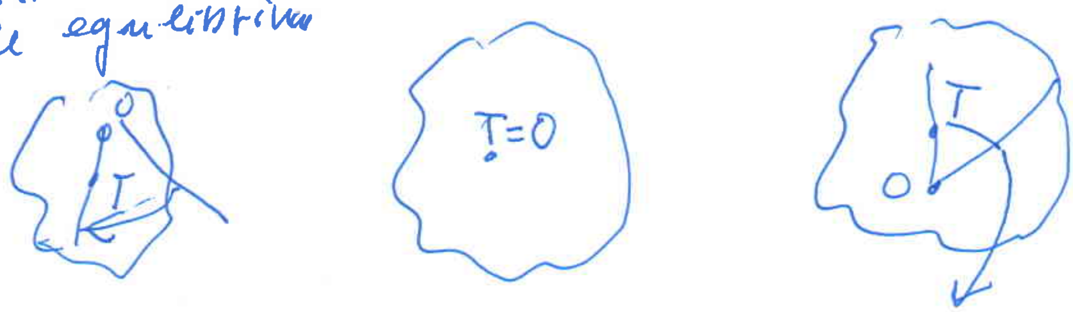
moment of the couple forces = arm cross one force

27. Equilibrium of rigid body

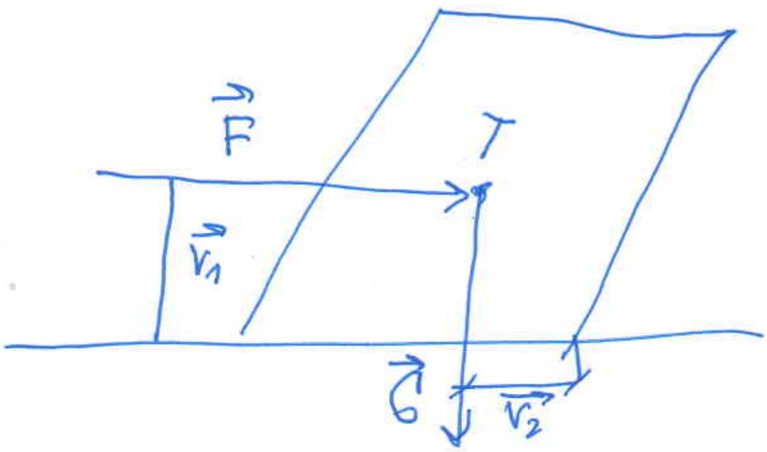
fixed location
stálá (stabilní)
stable equilibrium

indifferent position

labile position
instable eq.
[austerbed]



stability against overthrow



Turn over $\quad \vec{r_1} \times \vec{F_1} > \vec{r_2} > \vec{G}$
převrácení

The body is more stable if:

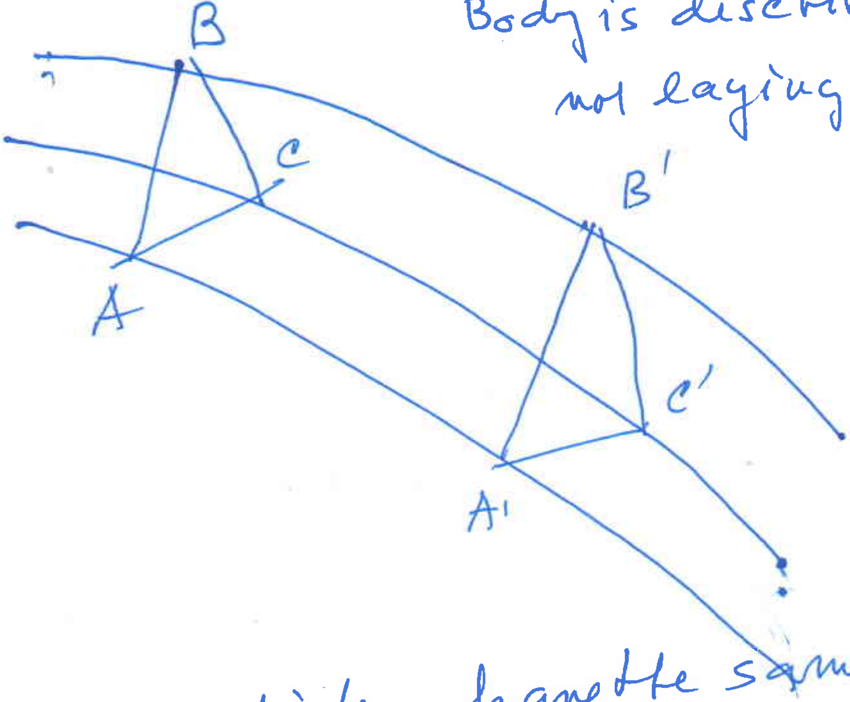
1, $\vec{G}$ when the mass is greater
2, $\vec{r_2}$ when the vector $\vec{r_2}$ is larger
3, $\vec{r_1}$ when the vector $\vec{r_1}$ is smaller

28. Motion of rigid body

Translation motion

Posuvný pohyb
(Translační)

Body is describe by three points not laying in one line

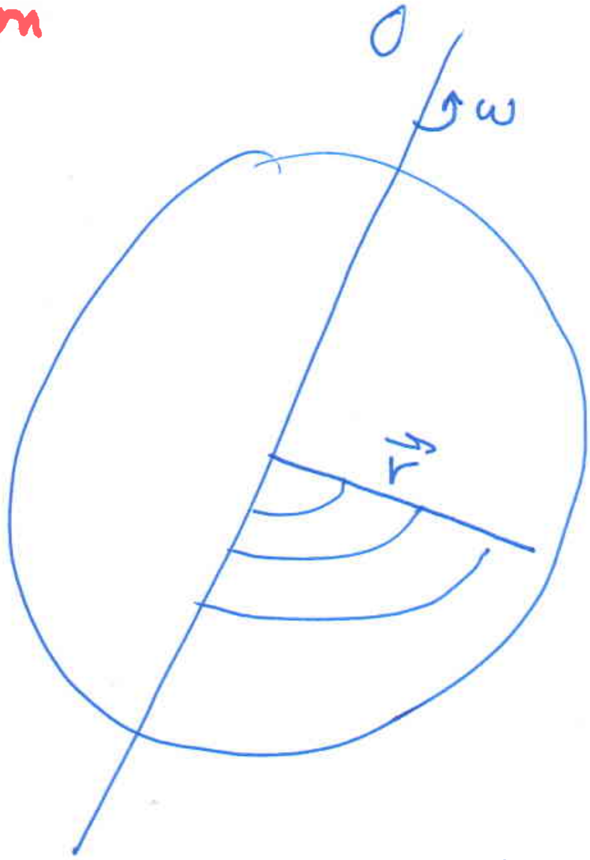


all particles have the same trajectorys and have at the same time the same velocity and acceleration.

To describe motion we can use only one point (center of mass)

Rotational motion

otáčivý pohyb



all points have the same angular velocity ω. ω = constant

1, all points on the axis are in rest

2, Other point have the same angular velocity ω = constant

29. Kinetic energy of rigid body

The kinetic energy of a rigid body is equal to the kinetic energy of all its points

$$\boxed{E_k = \frac{1}{2}\sum_{i=1}^{n} m_i v_i^2}$$

Translational motion

$v_i = v$

$$E_k = \sum_{i=1}^{n} \frac{1}{2} m_i v_i^2 = \frac{1}{2} m v^2$$

$$\boxed{E_k = \frac{1}{2} m v^2}$$

Rotational motion

$$E_k = \frac{1}{2} \sum m_i v_i^2 \qquad v = r \cdot \omega$$

$$E_k = \frac{1}{2} \boxed{\sum m_i r_i^2} \cdot \omega^2 \qquad \boxed{\omega_i = \omega}$$

$$\boxed{I = \sum_{i=1}^{n} m_i r_i^2}$$ = Moment of inertia of system of particles

Rigid body

$$\boxed{I = \int r^2 dm} \quad [m^2 kg]$$

continuous mass $\Rightarrow \int$

Kinetic energy (rotational motion)

$$\boxed{E_k = \frac{1}{2} I \omega^2}$$

Kinetic energy of rigid body

$$\boxed{\boxed{E_k = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2}}$$ König law proposition

30. moment of inertia

If $\frac{d\vec{b}}{dt} = \vec{M} = 0 \Rightarrow \vec{b} = \text{constant}$

$\vec{b} = \vec{r} \times \vec{p}$ $\qquad v = r.\omega$

$|\vec{b}| = |r.\ mv| = |r.m.r.\omega| = |m\ r^2\omega| = \text{constant}$

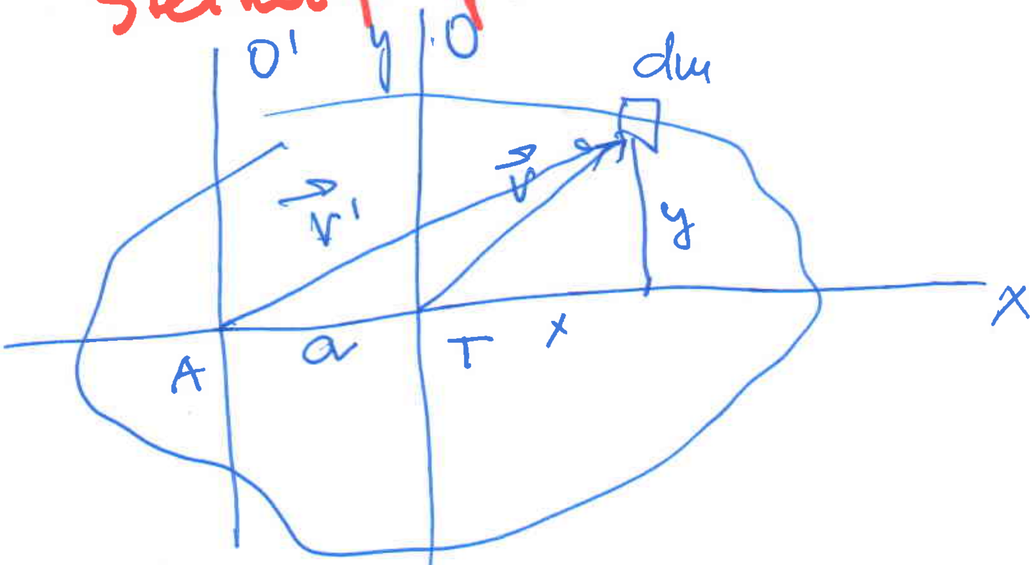
Figure skater ~~piruet~~ pirouette (piruet)

when he puts his arms together, he spins

When they open up $\Rightarrow$ he stops.

but $W = \Delta E = \frac{1}{2} I_1\ \omega_1^2 - \frac{1}{2} I\ \omega_2^2$

Steiner proposition



$$J_T = \int r^2 dm = \int (x^2+y^2) dm$$

Moment of Inertia with respect to axes O' which is paralel to axis O. O ∥ O'

$$J'_A = \int \vec{r}'^2 dm = \int [(x+a)^2 + y^2] dm$$

$$= \int (x^2 + 2ax + a^2 + y^2) dm$$

$$J' = \int (x^2+y^2) dm + 2a\int x\, dm + \int a^2 dm$$

$= 0 \Rightarrow x^* = \dfrac{\int x\, dm}{\int dm}$

masscenter T = 0

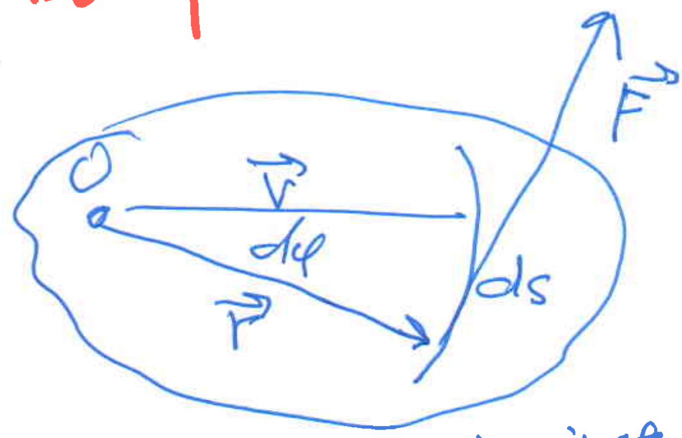
$$\boxed{J' = J_T + a^2 m}$$

Steiner proposition

31. Work and power by circular motion of rigid body

Scalar

$dW = F \cdot ds = F \cdot r \cdot d\varphi$



Work

$dW = M \cdot d\varphi$

It will change kinetic energy

The trajectory is circle [se: ke]

$E_k = \frac{1}{2} J \omega^2$

$dE_k = \frac{1}{2} \cdot J \cdot 2 \omega \, d\omega = J \omega \, d\omega$

Work = change of energy

$M \, d\varphi = J \omega \, d\omega \quad / \frac{1}{dt}$

$M \frac{d\varphi}{dt} = J \cdot \omega \frac{d\omega}{dt}$

$M \cancel{\omega} = J \cdot \cancel{\omega} \cdot \varepsilon$

$$\boxed{\vec{M} = J \cdot \vec{\varepsilon}}$$

Eq. of motion by rotation around a fixed axis

Power by rotation of rigid body around a fixed axis

$P = \frac{dW}{dt} = \frac{M d\varphi}{dt} = M \cdot \vec{\omega}$

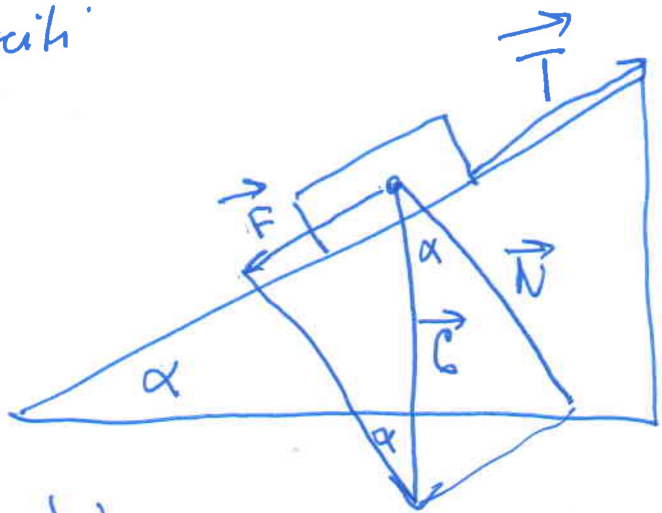
$$\boxed{P = \vec{M} \cdot \vec{\omega}}$$ [W]

32. Sliding friction

Friction
- rolling (valivé)
- sliding (smykové)
  - static α_0, μ_0
  - kinetic α, μ

Depends - material
- surface size
- relative velocity
- temperature

Inclined plane
naklonená rovina



Just before breaking away
$\vec{F} = \vec{T}$ $\qquad T = \mu N$

We must increase magnify the angle as long as the body starts move down.

$\vec{F} = T$

$$G \cdot \sin\alpha_0 = \mu_0 \cdot N = \mu_0 \cdot G \cdot \cos\alpha_0$$

$$\mu_0 = \frac{\sin\alpha_0}{\cos\alpha_0}$$

$$\boxed{\mu_0 = \tan\alpha_0}$$

Kinetic friction α, μ

When the body starts to move down then we can reduce the angle "α" so the body move down with constant velocity. v = constant $\Rightarrow F_{down} = 0$

$$F_{down} = F - T$$

$$0 = G \cdot \sin\alpha - \mu \cdot G\cos\alpha$$

$$\boxed{\mu = \operatorname{tg}\alpha}$$

materials	μ_0	α_0	μ	α
steel on steel	0,7	1°36'	0,6	48'
wood on wood	0,6	32°	0,5	26°