Dynamic of particle
8. Newton's laws of motion

Galileo Galilei , Philosoper and mathematician [matematišen]

Pisa 1564–1642
Padova

1. Improved the telescope [Impröved teliscop]
2. 1. newton law of inertia
3. The Earth orbits the Sun → prison [hamp.]
4. Trajectory bei acceleration depends $\sim t^2$

Johannes Kepler

Stuttgart 1571–1630

1600 came to Prague and he became an assistant to Tycho de Brahe and after his death → caesar mathematician and astrologer [astrolödžr]

Formulated:

1. Planets move around the sun in elliptical orbits.
   The Sun is in focus.
2. The area of positions vector over time are constant

$S_1 = S_2$ (t)

3. Ratio of $\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$

Isaac Newton

London 1643 – 1727

1687 Book "Mechanics" published

Formulated – The law of gravity

$$\vec{F} = \varkappa \cdot \frac{m_1 \cdot m_2}{r^2} \cdot \vec{r_0} \quad [N]$$

$$\varkappa = 6{,}67 \cdot 10^{-11} \left[\frac{N \cdot m^2}{kg^2} \right]$$

(intensity potential)

over a long distance

Newtons laws of motion

1. Law of inertia

Every body continues in its state of rest, or of straight line uniform motion, unless acted an external force.

2. Law of force $\quad \frac{d\vec{p}}{dt} = \vec{F}$

m .... mass

$\vec{p} = m \cdot \vec{v} \; [kg \frac{m}{s}]$ momentum

$\vec{F} = m \cdot \vec{a} \; [N]$ force

Rate of change of momentum is proportional to the applide force, and takes place in the direction of the force

$$\vec{F} = \frac{d(m\vec{v})}{dt} = m \cdot \frac{d\vec{v}}{dt} = m \cdot \vec{a}$$

3. Law of action and reaction

[vioeks]

action and reaction are equal and opposite direction

9. Solution of equation of motion

1. Basic problem of dynamics

We have trajectory

$$\boxed{\vec{r} = r_x \vec{i} + r_y \vec{j} + r_z \vec{k}}$$

we want to know force F?

a) $\vec{v} = \frac{d\vec{r}}{dt}$ — v_x, v_y, v_z

b) $\vec{a} = \frac{d\vec{v}}{dt}$ — a_x, a_y, a_z

c) $F_x = m \cdot a_x$

$F_y = m\, a_y$

$F_z = m \cdot a_z$

$$\boxed{\vec{F} = (F_x \vec{i} + F_y \vec{j} + F_z \vec{k})\ [N]}$$

2, Inverse problem

we know force $\boxed{\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}}$

we want to know trajectory $\vec{r}$?

$F_x = m a_x$

$a_x = \frac{F_x}{m}$ $\qquad\qquad a_y$

$\qquad\qquad\qquad\qquad a_z$

$a_x = \frac{dv_x}{dt}$

$\int dv_x = \int a_x dt$

$v_x = a_x \cdot t + v_{x0}$

$v_x = \frac{dr_x}{dt}$

$\int dr_x = \int v_x dt = \int (a_x t + v_{x0}) dt$

$r_x = \frac{1}{2} a_x t^2 + v_{x0} \cdot t + r_{x0}$

r_y

r_z

$$\boxed{\vec{r} = \left[\left(\frac{1}{2}a_x t^2 + v_{x0} t + r_{x0}\right)\vec{i} + (\quad)\vec{j} + (\quad)\vec{k}\right][m]}$$

I remind

1. Falling body – padající těleso

Free fall = volný pád

$g = 9{,}81 \frac{m}{s^2}$

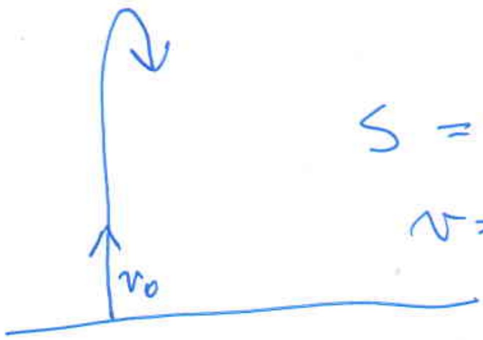
gravity acceleration

$$s = h - \frac{1}{2} g t^2$$

$$v = g t$$

2) Motion in vertical direction { up / down

Vertical throw

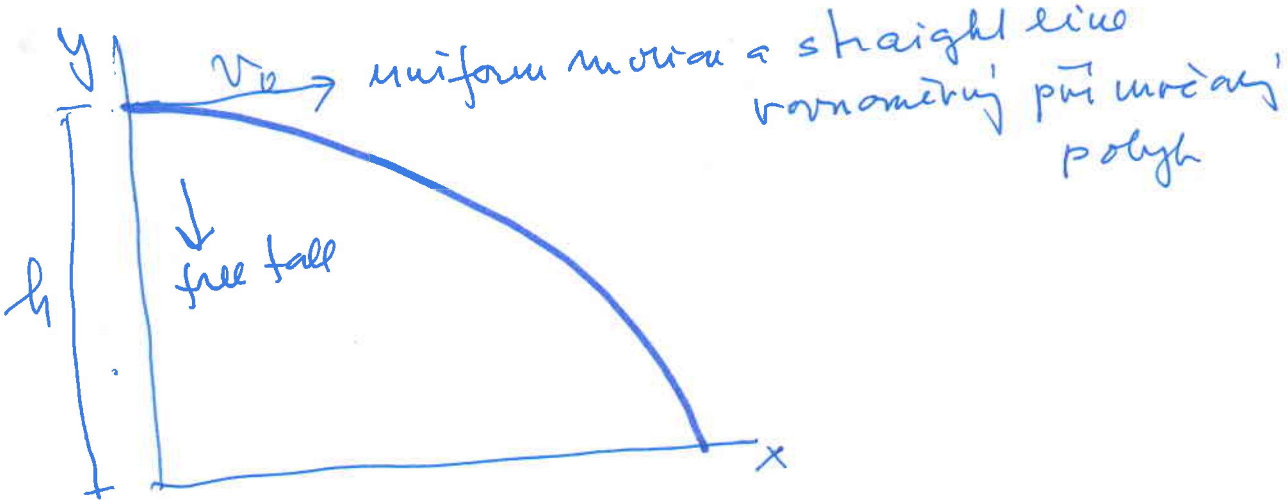


$$s = v_0 \cdot t - \frac{1}{2} g t^2$$

$$v = v_0 - g t$$

3, Horizontal motion

Horizontal throw



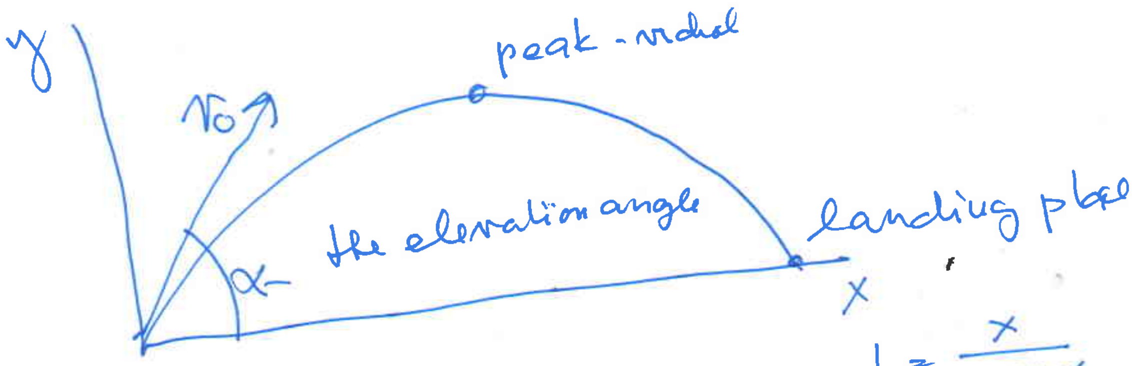
$$x = v_0 \cdot t \longrightarrow t = \frac{x}{v_0}$$

$$y = h - \frac{1}{2} g t^2$$

$$y = h - \frac{1}{2} g \cdot \frac{x^2}{v_0^2} \quad \text{(parabol)} \ [y, x^2]$$

4, Projection at an angle

šikmý vrh

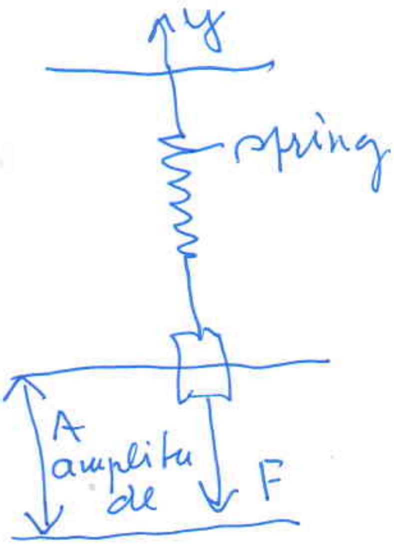


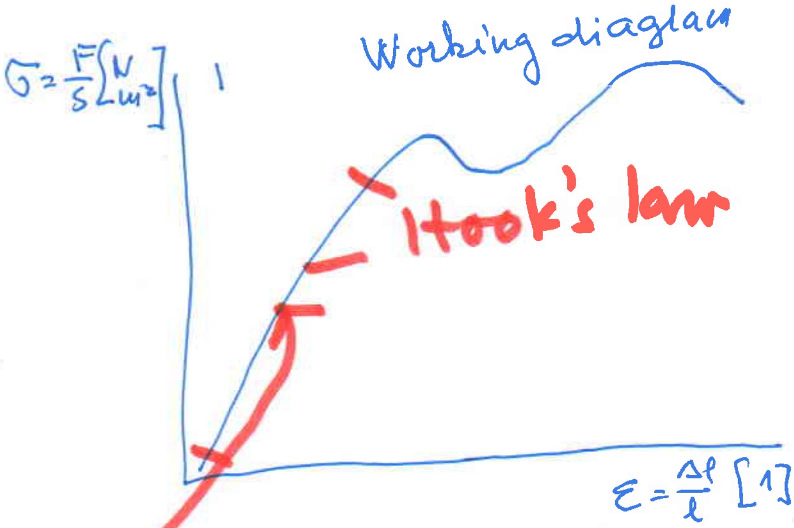
$$t = \frac{x}{v_0 \cos\alpha}$$

$$x = v_0 \cdot t \cdot \cos\alpha$$

$$y = v_0 \cdot t \sin\alpha - \frac{1}{2} g t^2$$

10. Linear harmonic oscillator





$\vec{F} = = -k\vec{r}$

$\boxed{F = -ky}$ **Force equation**

$k = -\frac{F}{y}$ spring constant
solidity of spring

theoretical force for elongation 100%

$my'' = -ky \;/\; \frac{1}{m}$

$y'' + \underbrace{\frac{k}{m}}_{\omega^2} y = 0$

$\omega^2 = \frac{k}{m}$ $\qquad \omega = \frac{2\pi}{T}$

$T = 2\pi \cdot \sqrt{\frac{m}{k}}$

$\boxed{y'' + \omega^2 y = 0}$

Differential eq. that describe free harmonic oscillations.

y'' – diferential eq. of secon order

$1, \omega^2$ – with constant coefficient

$= 0$ .... homogenous

$$y'' + \omega^2 y = 0$$

The solution will be of the form

$y = C \cdot e^{\lambda t}$ — C ... ~~c~~ constant, λ - the root of the characteristic equation

$y' = C \cdot \lambda \cdot e^{\lambda t}$

$y'' = C \cdot \lambda^2 \cdot e^{\lambda t}$

$$C \cdot \lambda^2 e^{\lambda t} + \omega^2 \cdot C \cdot e^{\lambda t} = 0$$

$$C \cdot e^{\lambda t} (\lambda + \omega^2) = 0$$

reprimand = vyttenout

$$\lambda_{1,2} = \pm j\omega$$

$\sqrt{-1} = \vec{j}$

the square root of negative one

Solution

$$y = C_1 e^{+j\omega t} + C_2 e^{-j\omega t}$$

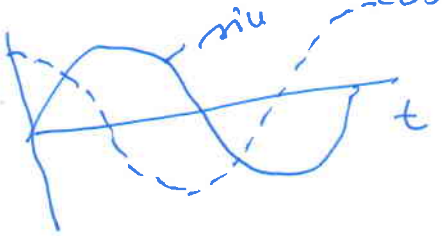
Eulerov vztah $\quad e^{\pm j\omega t} = \cos \omega t \pm \vec{j} \cdot \sin \omega t$

$$y = C_1 \cos(\omega t) + C_1 j \sin(\omega t) + C_2 \cos(\omega t) - C_2 j \sin(\omega t)$$

$$y = (C_1 + C_2)\cos(\omega t) + \underbrace{(C_1 - C_2)\vec{j} \cdot \sin(\omega t)}_{\text{imaginar path}}$$

$t=0 \quad (C_1 + C_2) = A$ from initial condition

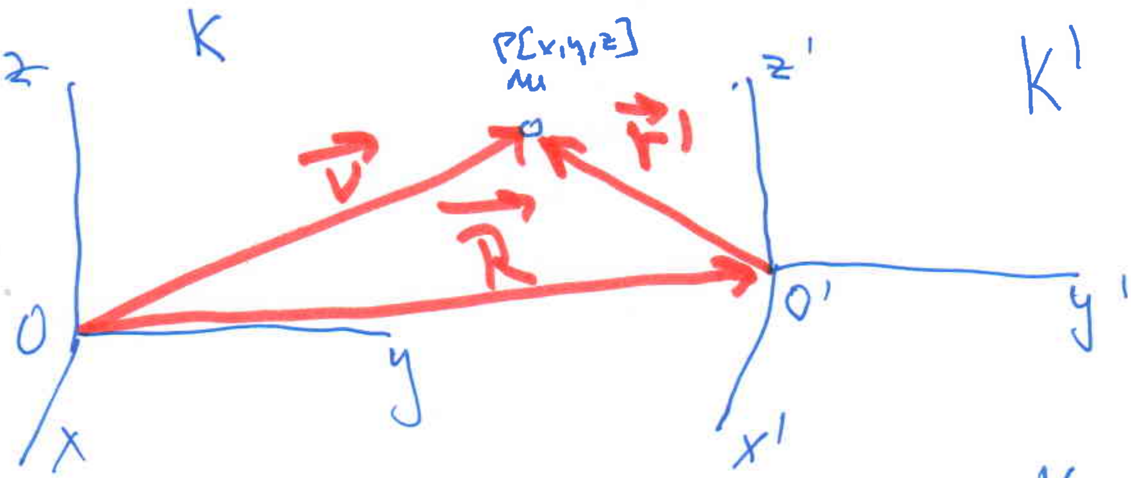
$$y = A \cdot \cos(\omega t)$$



$$\boxed{y = A \cdot \sin(\omega t + \varphi)}$$

Equation of displacement

11. Motion in noninertial frame works
[inerciální]



$\vec{r}$ .. position vector of point P relative to system K

$\vec{r}'$ .. —"— —"— —"— —"— K'

$\vec{R}$ = vector of reciprocal displacement
[vzájemné]

$$\vec{R} = \vec{r} - \vec{r}'$$

$$\vec{r}' = \vec{r} - \vec{R}$$

$$\frac{d\vec{r}'}{dt} = \frac{d\vec{r}}{dt} - \frac{d\vec{R}}{dt}$$

$$\vec{v}' = \vec{v} - \vec{u}$$

$\vec{v}' = \frac{d\vec{r}'}{dt}$ .. relative velocity of mass point to framework K'

$\vec{v}$ .. absolute velocity of mass point to fram. K

$\vec{u} = \frac{d\vec{R}}{dt}$ .. drift velocity
unášivá rychlost

A) Let's $\boxed{\vec{u} = \text{constant}}$

$\Rightarrow$ $\boxed{K' \text{ is inertial}}$ $\qquad \vec{v}' = \vec{v} - \vec{u}$

$$\frac{d\vec{r}'}{dt} = \frac{d\vec{r}}{dt} - \vec{u} \quad \Big/ \cdot dt$$

$$\int d\vec{r}' = \int d\vec{r} - \int \vec{u}\, dt$$

$$\boxed{\vec{r}' = \vec{r} - \vec{u} \cdot t}$$

Galileau's transformations equations

$$\left.\begin{aligned} x' &= x - u_x \cdot t \\ y' &= y - u_y \cdot t \\ z' &= z - u_z \cdot t \end{aligned}\right\}$$

We can convert $K \rightarrow K'$, $K' \rightarrow K$

$$\frac{d\vec{v}'}{dt} = \frac{d\vec{v}}{dt} - \frac{d\vec{u}}{dt}$$

but $\frac{d\vec{u}}{dt} = 0$, $\vec{u} = \text{constant}$

$$\vec{a}' = \vec{a} \quad \Big/ \cdot m$$

$$m\vec{a}' = m\vec{a}$$

$$\vec{F}' = \vec{F}$$

If system K' is moving uniform and straight-line considering to system K, holds in both frameworks the same moving equations.

2, Let $\vec{u} = \vec{u}(t)$

$\Rightarrow$ Framework K' is not inertial

$$\frac{d\vec{v}'}{dt} = \frac{d\vec{v}}{dt} - \frac{d\vec{u}}{dt}$$

$\frac{d\vec{v}'}{dt} = \vec{a}'$ ... relative acceleration considering to K'

$\frac{d\vec{v}}{dt} = \vec{a}$ .. absolute acc. considering to system K

$\frac{d\vec{u}}{dt} = \vec{a}_u$ .. drift acceleration of framework K' considering to fram. K

$$\vec{a}' = \vec{a} - \vec{a}_u \quad \cdot | m$$

$$m\vec{a}' = m\vec{a} - m\vec{a}_u$$

$$\boxed{\vec{F}' = \vec{F} + \vec{F}_s}$$ eq. for force

when $\boxed{\vec{F}_s = -m\vec{a}_u \text{ is a inertial force}}$

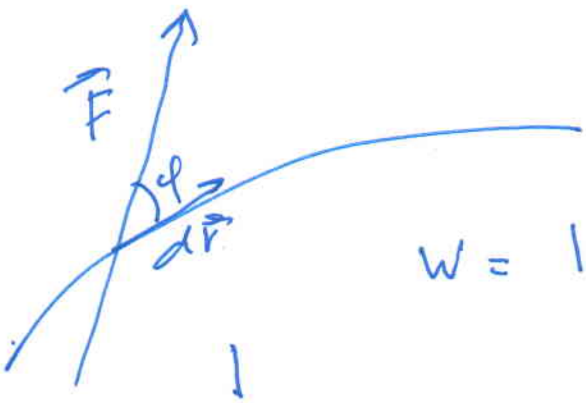
If we solve dynamic problem in framework K' which moves with drift acceleration $\vec{a}_u$ we have to add inertial the forces of reciprocal action force $\vec{F}_s = -m\vec{a}_u$.

For example: The bus – by starting
by stopping (halting)
by curving

Watcher on pavement wonder what happen?

12. Work, power [I remind you]

Work $W = \int \vec{F} \cdot d\vec{r}$ $[N \cdot m = J]$ joule



$$W = |\vec{F}| \cdot |d\vec{r}| \cdot \cos\varphi$$

Power

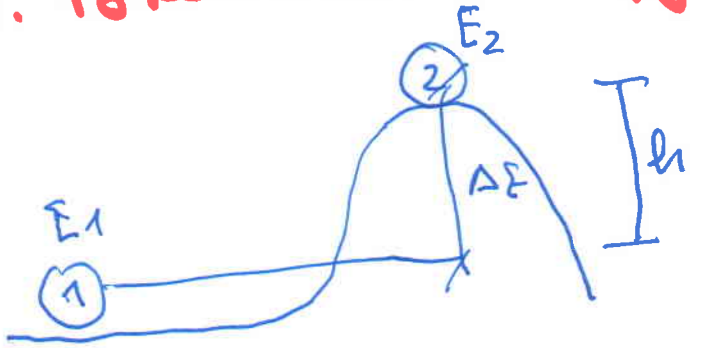
average power $\bar{P} = P_s = \frac{\Delta W}{\Delta t}$ $\left[\frac{J}{s} = W\right]$ watt

Instantaneous power

$$P = \frac{dW}{dt} \; [W]$$

$$P = \frac{d(\vec{F} \cdot d\vec{r})}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$

13. Potential energy



E – energy dependes on position / velocity

$$W = \vec{F}\cdot d\vec{r} = \int_0^h m\cdot a\cdot d\vec{r}$$

Earth $a = g = 9{,}81\ \frac{m}{s}$ gravitation acceleration

$$W = E_p = \int_0^h m g\, dr = mg[r]_0^h = mgh$$

$$\boxed{E_p = mgh \quad [J]}$$ Potentional energy

$$E_k = W = \int_{\vec{v}_1}^{\vec{v}_2} \vec{F}\, d\vec{r} = \int_{\vec{v}_1}^{\vec{v}_2} (F_x dx + F_y dy + F_z dz) =$$

$$F_x = m a_x = m\cdot\frac{dv_x}{dt}$$

$$E_k = m\int\left(\frac{dv_x}{dt}\cdot dx + \frac{dv_y}{dt}dy + \frac{dv_z}{dt}\cdot dz\right)$$

$$E_k = m\cdot\int_{\vec{v}_1}^{\vec{v}_2} v_x dv_x + v_y dv_y + v_z dv_z) =$$

$$E_k = \frac{1}{2}m\left[v_x^2 + v_y^2 + v_z^2\right]_{\vec{v}_1}^{\vec{v}_2} = \frac{1}{2}m(v_2^2 - v_1^2)$$

If $v_1 = 0 \Rightarrow$ $$\boxed{E_k = \frac{1}{2}mv^2\ [J]}$$ Kinetic energie

15. The law of conservation of mechanical energy

$$\boxed{E_k + E_p = \text{constant}}$$

Christiaan Huygens
1629 - 1695
German

for conservative forces — gravitation force
— elastic forces

For disipative forces — friction

Energy can be changed from one form to another, but it cannot be lost

Friction — some parts of energy converted into heat energy

1a Impulse of force

$\vec F = \frac{d\vec p}{dt}$

$\vec F \cdot dt = d\vec p$

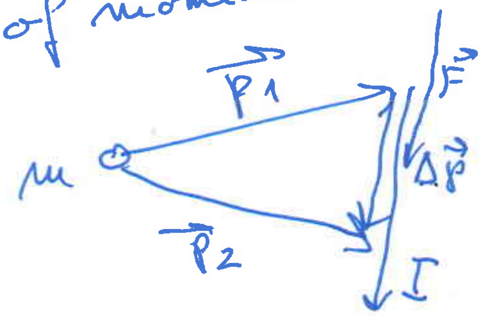
$$\vec I = \int_{t_1}^{t_2} \vec F \cdot dt = \int_{\vec p_1}^{\vec p_2} d\vec p = \vec p_2 - \vec p_1 = d\vec p$$

$$\boxed{\vec I = \int_{t_1}^{t_2} \vec F\, dt = \vec F (t_2 - t_1)}$$ – time effect of force

$\vec I$ ... impulse of force

$$\boxed{\vec I = \vec p_2 - \vec p_1 = \Delta \vec p}$$ – momentum theorem [sitrom]

Impulse of force is equal to change of momentum

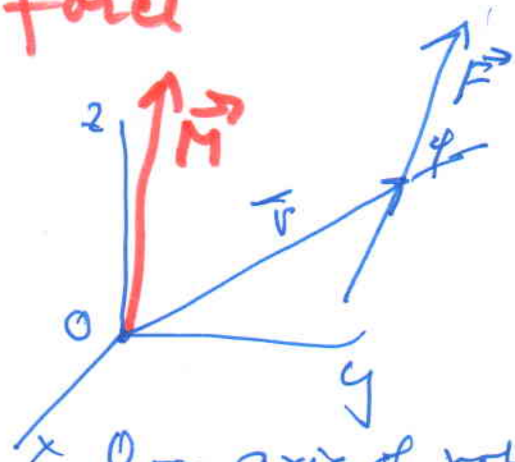


If $\boxed{\vec F = 0}$ $\left(\vec F = \frac{d\vec p}{dt}\right)$, than $\boxed{\vec p = \text{constant}}$

1LN

17. Moment of force

$$\boxed{\vec{M} = \vec{r} \times \vec{F}}$$



O ... axis of rotation

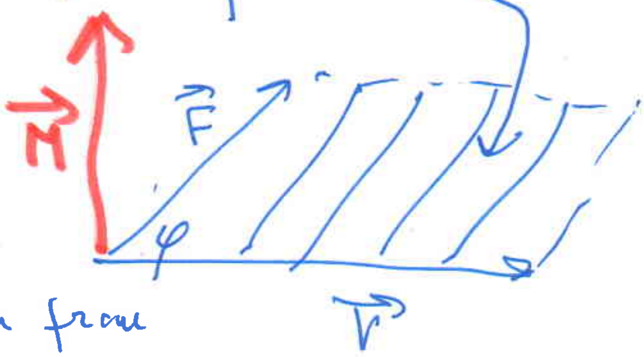
$\vec{r}$ .... arm

Torque [to:k]

kroutící síla (točivý moment)

Amplitude

$$|\vec{M}| = |\vec{r}| \cdot |\vec{F}| \cdot \sin\varphi$$



the right-hand-rule

If fingers show rotation from $\vec{r}$ to $\vec{F}$, than thumb schow direction of vector $\vec{M}$.

Determinant
[diterminent]

$$\vec{M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} =$$

$$= \vec{i}\, y F_z + \vec{j}\, z F_x + \vec{k}\, x F_y - \vec{k}\, y F_x - \vec{j}\, x F_z - \vec{i}\, z F_y$$

Rotational impuls

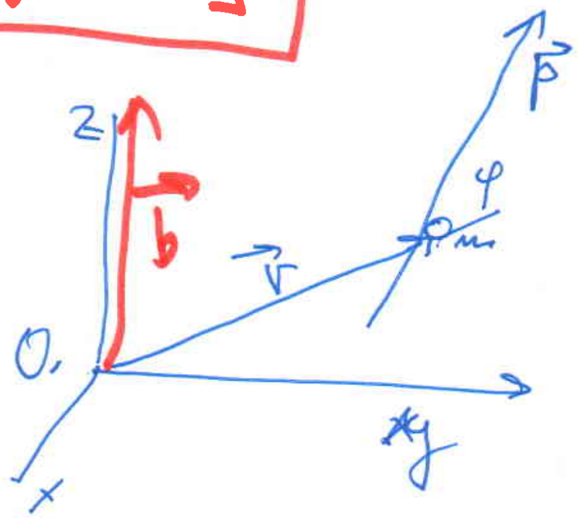
$$\boxed{\vec{L} = \int_{t_1}^{t_2} \vec{M}\, dt} \quad [m.N.s]$$

Impuls of moment of force

18. Moment of momentum

$$\boxed{\vec{b} = \vec{r} \times \vec{p} \ [kg \cdot m^2 s^{-1}]}$$

angular momentum



19. Continuity between moment of force and moment of momentum

$$\vec{M} = \vec{r} \times \vec{F} \qquad \vec{F} = \frac{d\vec{p}}{dt}$$

$$\vec{b} = \vec{r} \times \vec{p} \qquad \vec{p} = m\vec{v}$$

$$\frac{d\overbrace{(\vec{r} \times m\vec{v})}^{\vec{b}}}{dt} = \left(\frac{d\vec{r}}{dt} \times m\vec{v}\right) + \left(\vec{r} \times \frac{d(m\vec{v})}{dt}\right)$$

$$= \underbrace{(\vec{v} \times m\vec{v})}_{\substack{\vec{v} \parallel m\vec{v} \\ 0}} + \underbrace{(\vec{r} \times \vec{F})}_{\vec{M}}$$

$$\boxed{M = \frac{d\vec{b}}{dt}}$$

II. second equation of motion

If $\vec{M} = 0$ from it follows, that

$\vec{b} =$ constant.

For example: Rotation of the Earth

Translational motion		Rotational motion	
Force	$\vec{F} = m \cdot \vec{a}$	Moment of force	$\vec{M} = \vec{r} \times \vec{F}$
Momentum	$\vec{p} = m \cdot \vec{v}$	moment of momentum	$\vec{b} = \vec{r} \times \vec{p}$
Impuls of force	$\vec{I} = \int \vec{F} dt = d\vec{p}$	Rotational impulse	$\vec{L} = \int \vec{M} dt = d\vec{b}$
I. equation of motion	$\vec{F} = \frac{d\vec{p}}{dt}$	II. eq. of motion	$\vec{M} = \frac{d\vec{b}}{dt}$